KINEMATIC DEFORMATION ANALYSIS OF THE FIRST ORDER BENCHMARKS IN THE NETHERLANDS

Arnoud de Bruijne[1], Frank Kenselaar[2] and Frank Kleijer[2]

[1]*Survey Department, Ministry of Transport, Public Works and Water Management, PO Box 5023, 2600 GA Delft, The Netherlands (a.j.t.bruijne@mdi.rws.minvenw.nl)*

[2]*Delft University of Technology, Department of Mathematical Geodesy and Positioning, Thijsseweg 11, 2629 JA Delft, The Netherlands (f.kenselaar@geo.tudelft.nl)*

Abstract

The Netherlands have a large record of precise levelling networks, enabling us to monitor vertical deformations on a national scale. The Survey Department of the Ministry of Transport, Public Works and Water Management has selected four sets of primary and several sets of secondary levelling campaign networks between 1926 and 2000 to analyze the vertical movements of the first order benchmarks that constitute the Dutch height datum NAP. In co-operation with Delft University of Technology, a strict kinematic deformation analysis procedure has been developed and implemented in software that allows us to compute benchmark velocities.
Since the primary levelling campaigns were measured in relatively large time spans, significant movements can be expected within an epoch. Therefore, the choice was made for a strict kinematic approach where each levelling observation has its individual time label. Canceling epochs, however, severely complicates the data organization and estimability of the problem. Combined with the large amount of data, network reduction and non-standard estimation algorithms were required. The paper discusses the model and the estimation and testing procedure, and demonstrates first analysis results.

1. Introduction

The estimation of vertical movements of the land is extremely important in The Netherlands, because large parts are situated below sea level. To achieve this, measurements for the Dutch height datum NAP (*Normaal Amsterdams Peil*) are analyzed. The backbone of the NAP is formed by about 220 underground benchmarks, which are well founded in ancient Pleistocene sands. Furthermore, there are about 50.000 regular (surface) benchmarks.
As early as the year 1875, the first nationwide primary levelling campaign was started. For several reasons (e.g. lack or loss of benchmarks) successive primary and secondary levelling campaigns were conducted. After the third primary levelling, suspicion arose about the stability of the Pleistocene sands. This was confirmed by the fourth primary levelling campaign: the underground benchmarks displayed a significant vertical movement. The northwest of the country subsides with respect to the southeast. This instability of the backbone of the Dutch height datum, with velocities in the order of one millimeter per year, was the main reason for conducting the fifth primary levelling campaign, see figure 1. The aim was now to model the vertical deformation of The Netherlands in a strictly kinematic sense, apart from the conventional task to maintain the NAP. This paper discusses the kinematic model that will be applied, including first test results.
Section 2 of this paper describes the data and the necessity of strict kinematic modelling. The kinematic model itself is discussed in section 3, while section 4 deals with the estimation and testing procedure. Section 5 discusses specific estimation problems when analyzing the first order (underground) benchmarks. A simple example is treated in section 6, followed by a preliminary discussion of the application of the model to the primary levelling networks in section 7. The paper is concluded by some remarks about further research in section 8.

Figure 1: 5th first order levelling network.

2. Large scale vertical deformation analysis from the primary benchmarks

The large record of precise levelling data in The Netherlands consists of the second to fifth primary levelling networks (the data of the first primary levelling is not available digitally). It also contains three secondary levelling data sets, of which the third one is currently being measured. The corresponding campaign dates and total levelling lengths of the networks are listed in table 1.

Levelling data	year	length
2nd primary set	1926-1940	4.592 km
3rd primary set	1950-1959	4.600 km
4th primary set	1965-1978	4.521 km
1st secondary set	1976-1986	22.261 km
2nd secondary set	1987-1996	29.000 km
5th primary set	1996-1999	6.200 km
3rd secondary set	1998-2007	29.000 km

Table 1: Primary and secondary levelling campaigns in The Netherlands.

Note that the campaign dates cover relatively large time spans, resulting in significant deformation to be expected within an epoch. This is why we want to apply a strict kinematic modelling, where each levelling observation has its individual time label. However, the data has never been gathered with a view to be processed using such an approach. This significantly complicates the data organization and the ability to estimate vertical movements of the benchmarks. In order to stay on the strict side of kinematic modelling, reduction of the large amount of data, as well as new algorithms were required.

As an indication, a non-strict kinematic computation was carried out in 1999, when the total (although raw) data set (4 first order and 2 second order sets) became first available. This again confirmed the findings of earlier comparisons. However, this time more details could be seen, like a significant subsidence region in the southeast. Some recommendations after this indicative computation were:

- The data set must be free of errors that can be detected in a single epoch network analysis;
- Strict kinematic modelling must be aimed for;
- A priori reduction of the data minimizes the complexity of the analysis.

3. The strict kinematic model

In a kinematic vertical deformation model, the height $H_{i,t}$ of a benchmark i at time t is written as

$$H_{i,t} = H_{i,t_0} + z_{i,t-t_0} \,, \tag{1}$$

with H_{i,t_0} the benchmark height at a specified time t_0 and $z_{i,t-t_0}$ a temporal function of the benchmarks' movements since t_0.

In principle both the temporal function z and t_0 can be chosen differently per benchmark. Often, polynomial functions are considered. In VERHOEF AND DE HEUS 1995, polynomials are estimated from benchmark heights, including discontinuities or breakpoints. If applicable, one could also consider a spatial-temporal function for z, assuming a relation between benchmark behavior at different locations (see e.g. KENSELAAR AND QUADVLIEG 2001). In this case there is no sensible vertical deformation model to start with. On the contrary, we would like to gain insight in large-scale long-term vertical deformations of the Netherlands subsoil from analyzing the spatial pattern of individual benchmark movements. Therefore we start with a very simple model, assuming a constant individual velocity per benchmark, i.e.

$$z_{i,t-t_0} = v_i (t - t_0) \,, \tag{2}$$

with v_i the velocity term per benchmark (e.g. in mm/year).

The choice of t_0 is completely arbitrary and only influences the benchmark height H_{i,t_0}. By adopting January 1, 2000 for t_0 (the end of the period of data available) the estimated heights H_{i,t_0} can be interpreted as actual benchmark heights.

In this study we opt for a strict kinematic analysis approach. It is assumed that the levelling measurements cannot be gathered in epochs, sharing the same time label. In this case epoch network analysis is impossible and the vertical deformation model (2) cannot be obtained from the benchmark heights in (1). The deformation model must be determined by straightforward estimation from the original levelling observations of the primary and secondary levelling campaigns.

Using (1) and (2), the observation equation of a levelled height difference $h_{ij,t}$ between benchmarks i and j at time t, can be written as

$$\begin{aligned} \underline{h}_{ij,t} &= -H_{i,t} + H_{j,t} + \underline{e}_{ij,t} \\ &= -H_{i,t_0} + H_{j,t_0} - v_i (t - t_0) + v_j (t - t_0) + \underline{e}_{ij,t} \;. \end{aligned} \tag{3}$$

Equation (3) is made fit with the unknown residual term $\underline{e}_{ij,t}$, accounting for the stochastic discrepancies between data and model. The residuals are assumed to have zero expectation and a standard deviation

$$\sigma_{h_{ij,t}} = a + b\sqrt{l_{ij}} + c\, l_{ij} \,, \tag{4}$$

where coefficients a (in mm), b (in mm/√km) and c (in mm/km) can be specified per levelling observation. In this way, flexibility is offered for the determination of the stochastic model. For spirit levelling usually just a factor b is chosen. For very short levelling lines a factor a is required, while a factor c could be useful for hydrostatic levelling.
The assumption of linear benchmark velocity in the vertical deformation model is likely to appear a too simple approximation of the benchmark behavior. It may therefore be expected that the residuals in (3) cannot be explained by the levelling measurement precision in (4) only. As will be briefly reviewed in the following section, statistical testing will be applied to check whether the model is valid and to suggest adaptations in the model or data.

Equation (3) shows that from levelled height differences, only height *differences* ($H_{ij,t_0} = H_{j,t_0} - H_{i,t_0}$) and velocity *differences* ($v_{ij} = v_j - v_i$) between benchmarks can be estimated. In order to obtain a solution, one height and one velocity parameter have to be fixed at an arbitrary value. For instance, for one benchmark a height is chosen and the velocity is assumed zero. The estimated velocities of the other benchmarks must then be interpreted as *relative* to this constrained benchmark.
For the levelling data of all primary and secondary levelling campaigns under study, the linear model of observation equations, based on (3) and (4), can be written as

$$\underline{h} = \begin{pmatrix} A & TA \end{pmatrix} \begin{pmatrix} H_0 \\ v \end{pmatrix} + \underline{e} \; ; \quad Q_h \, , \tag{5}$$

with:
- h — vector of m levelled height differences with individual time labels;
- A — $m \times n$ matrix relating the levelled height differences and benchmark heights (levelling matrix, containing rows with one –1 and one 1, and zeros elsewhere);
- T — $m \times m$ diagonal matrix with elements $t - t_0$, where t is the time label of the corresponding observation;
- H_0 — vector of n unknown benchmark heights at time t_0;
- v — vector of n unknown benchmark velocities;
- e — vector of m residuals;
- Q_h — $m \times m$ diagonal variance matrix, with variances $\sigma^2_{ij,t}$ according to (4).

This *kinematic* levelling network model can be compared with an ordinary levelling network model (with the same time label assumed for all levelling observations), where the unknown benchmark heights are replaced by the kinematic model (1), (2). The (theoretical, as we will see later) rank deficiency of 2 is solved by adopting one benchmark as a base-point and eliminating its height and velocity from the vector of unknowns. The redundancy of the model is then $m-2(n-1)$.

4. Estimation of benchmark velocities from levelling data

Model (5) can be solved by least-squares adjustment. In order to allow computation of the large network on a PC, column minimum degree ordering and sparse matrix Cholesky decomposition techniques are applied, making use of the scanty number of non-zeros in the levelling matrix.
Optimal estimates for the benchmark heights and velocities are determined in a stepwise procedure of least-squares estimation, statistical hypothesis testing and adaptation of both the data and model. In each step the model and data - the actual null hypothesis model - are tested against a large number of alternative hypotheses, each suggesting a specific model adaptation or possible error(s) in the data. Presently the following types of alternative hypotheses are considered:

- *Observation tests* or datasnooping, sequentially testing all levelling observations for individual errors (m test statistics);

- *Point tests*, sequentially testing all benchmarks for significant deviation from the constant velocity assumption, without further specification (*n* test statistics);
- *Overall model test*, indicating whether the model and data match, without further specification (1 test statistic).

A first data analysis could supply reason to concern other types of alternative hypotheses. One could think of testing the significance of the estimated velocity, testing the significance of a specific more complex temporal function, or testing for a significant discontinuity in the benchmarks' behavior (see VERHOEF AND DE HEUS 1995).

As long as the null hypothesis is rejected, the most significant error or model adaptation is suggested by the largest test quotient (quotient of test quantity and critical value, rejected when larger than one). Adaptations can consist of mutations of the observation data or benchmarks considered, or adaptations in the stochastic model, e.g. changing the levelling standard deviation. The adapted model acts as null hypothesis in the next step of estimation and testing, until all tests are accepted. The testing procedure is treated in more detail in KENSELAAR 2001, and in DE HEUS ET AL. 1995.

In the previous description it was conveniently assumed that for each benchmark, appearing in any of the levelling campaigns under study, a velocity could be estimated. In practice this certainly will not be a valid assumption since the primary and secondary levelling campaigns were never designed for a strict kinematic analysis like this. In the campaigns static levelling networks were measured, although sometimes within a period of up to ten years, with the intention to (re)define the fundamental benchmark heights of the NAP height datum. Except for the fundamental benchmarks, the networks consist of many intermediate and tie points that are irrelevant for this vertical deformation analysis. It is obvious that no velocity can be determined for benchmarks that are measured by levelling lines with the same time label only. When the time lag is small, the accuracy of the estimated velocity can be very low, e.g. when all measurements stem from one levelling campaign. These (near) rank deficiencies make the problem practically incomputable and must be eliminated beforehand.

In principle, different strategies can be followed to handle this problem. One could think of

1. *Reduction* of the kinematic network by a search for (and elimination of) those benchmarks for which no velocity can be determined.
2. Restriction of the analysis to a suitable subset of the data by *selection* of measurements and benchmarks that can definitely be computed. One could e.g. start with only those benchmarks that are connected by at minimum two observations with sufficient time lag.
3. Extension of the model (5) with *pseudo observations* or soft constraints for the benchmark velocities. It is e.g. possible to add zero-valued observations for the *n* velocity terms. By giving these pseudo observations a large standard deviation (relative to the measurement precision), they will practically have no influence on the estimation, while the rank deficiencies are taken care of.
4. Application of linear algebra techniques for a *numerical determination* of the null space of matrix ($A \; TA$) in (5), and of a minimal set of velocity parameters that must be constrained to allow computation.

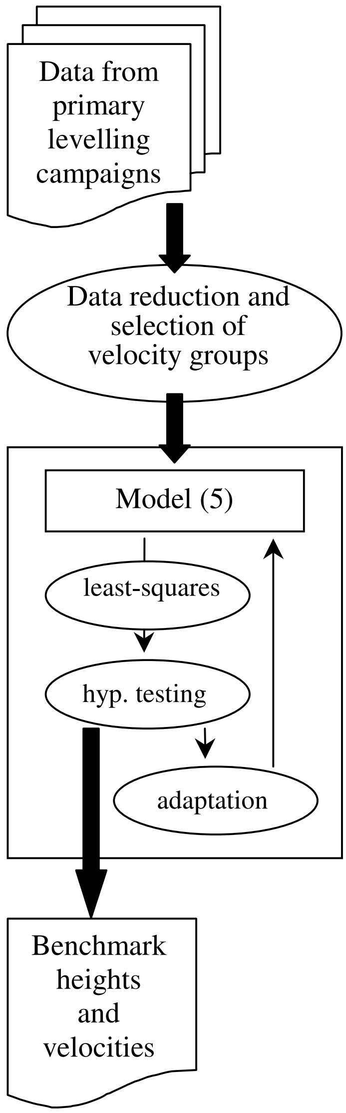

Figure 2: Processing scheme.

Each strategy has its advantages and drawbacks. At the moment a combination of reduction and selection is applied, and is briefly discussed in the next section. For a detailed discussion we refer to KLEIJER ET AL. 2001. Figure 2 presents an overview of the processing scheme.

5. Network reduction and selection of groups that share the same velocity constraint

For this long-term vertical deformation analysis the measurement time label can be rounded to months or even years. This could reduce the number of benchmarks that are only determined by measurements with a very small time lag.

The reduction is based on the connection of benchmarks (in topology and time) with the rest of the kinematic network, and consists of three steps:

1. Detection of points that are connected by only one observation (loose end). Both benchmark and observation are eliminated from the data set.
2. Detection of points that are connected by only two observations with the same time label. For these benchmarks clearly no velocity can be computed. The benchmark is eliminated while the two levelling lines are combined to a new observation, since they can still support the determination of other benchmarks.
3. Detection of points that are connected by only two observations, and have a neighbor with the same property. Such a pair of benchmarks is only determined by three observations, which is insufficient for determination of their height and velocity. Trajectories of two or more of such benchmarks and their observations are eliminated, since observations with different time labels cannot be combined.

After elimination of a benchmark, new candidates for reduction could show up. Therefore the reduction steps must be completed cyclically until no further elimination of benchmarks is obtained. In figure 3 the resulting kinematic network is shown, consisting of the 2nd to 5th primary levelling campaigns, after reduction. The number of observations is reduced from 27770 to 16132 and the number of benchmarks is reduced from 17948 to 6366.

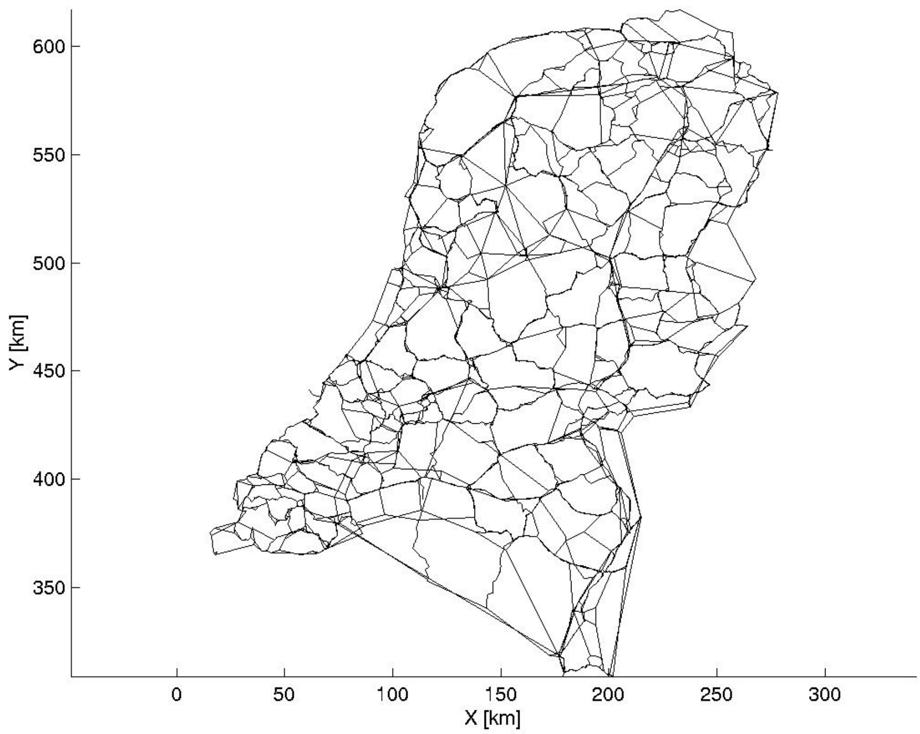

Figure 3: Reduced kinematic network of primary levelling campaigns (rounded to years).

After reduction the kinematic network could still suffer from a rank defect larger than two, since it is not guaranteed that all benchmark velocities can be computed with respect to one and the same velocity constraint, i.e. a fixed zero-velocity for one benchmark. Hereby we assume that all benchmarks are somehow connected as one topological network and one height constraint suffices. In the practice of the primary levelling campaigns a situation like in figure 4 could easily occur. The numbers refer to the different time labels of the levelling observations. This situation is underdetermined and benchmark heights and velocities can only be computed when both groups of points AB, respectively CDE, have their own velocity constraint. E.g., the velocity of point B is then computed with respect to point A (zero-velocity), while the velocities of points D and E are computed with respect to point C.

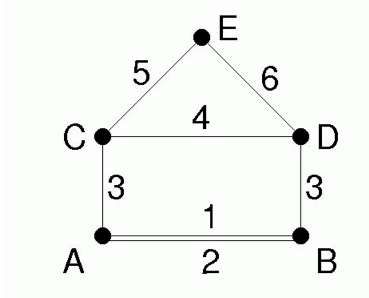

Figure 4: Kinematic network requiring two velocity constraints.

Along these lines algorithms have been developed that select 'velocity groups' of benchmarks that can be computed with respect to one and the same velocity constraint. The most important principle used is that two benchmarks, connected by at least two observations with different time label, can be considered a velocity group. By joining these points, their observations to other points redistribute and possibly other points can be joined. This process of velocity group selection by joining points topologically is demonstrated in figure 5.

In case the kinematic network is divided in several velocity groups the vertical deformation analysis can be performed per group. Estimated benchmark velocities can only be compared within the group and with respect to the velocity constraint.

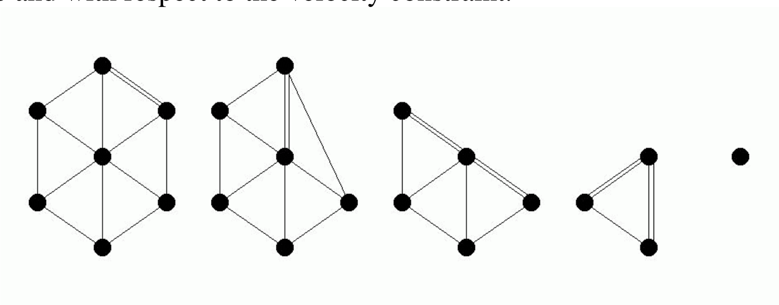

Figure 5: Joining points to recover a velocity group.

6. Example: the Roswinkel data set

This small data set consists of five levelling networks above an isolated gas field, respectively measured in 1980, 1985, 1990, 1994 and 1997. Within an epoch all measurements have the same time label. It is known that the gas extraction driven land subsidence follows a smooth bowl, with a maximum subsidence velocity of about 10 millimeter per year in its center. After elimination of benchmarks that appear in only one epoch, the kinematic network can be computed by fixing the height and velocity of just one benchmark, chosen far outside the subsidence bowl.

Since the first epoch was measured before gas extraction, the discontinuity in benchmark behavior around 1983 causes the overall model test of the first run to be rejected with a test quotient of 6.22. After omitting the 1980 epoch this test quotient reduces to 1.75. The largest test quotient now belongs to a point test and reads 1.89. After elimination of the corresponding benchmark the overall model again shows the largest test quotient, 1.56. The a priori standard deviation of 0.7

mm/√km seems too strict to fit our constant velocity kinematic model. Multiplication by 1.25 (√1.56) should make the testing accepted.
The resulting benchmark velocities are visualized in figure 6. The largest estimated velocity is 10.85 mm/year, belonging to a benchmark near the center of the subsidence bowl. The figure clearly shows the spatial coherence of the benchmark velocities, suggesting a spatial-temporal description of the subsidence would be appropriate. For this data set a 7-parameter spatial-temporal subsidence bowl (KENSELAAR AND QUADVLIEG 2001) has actually been estimated. Its ellipsoidal contour line is also drawn in the figure.

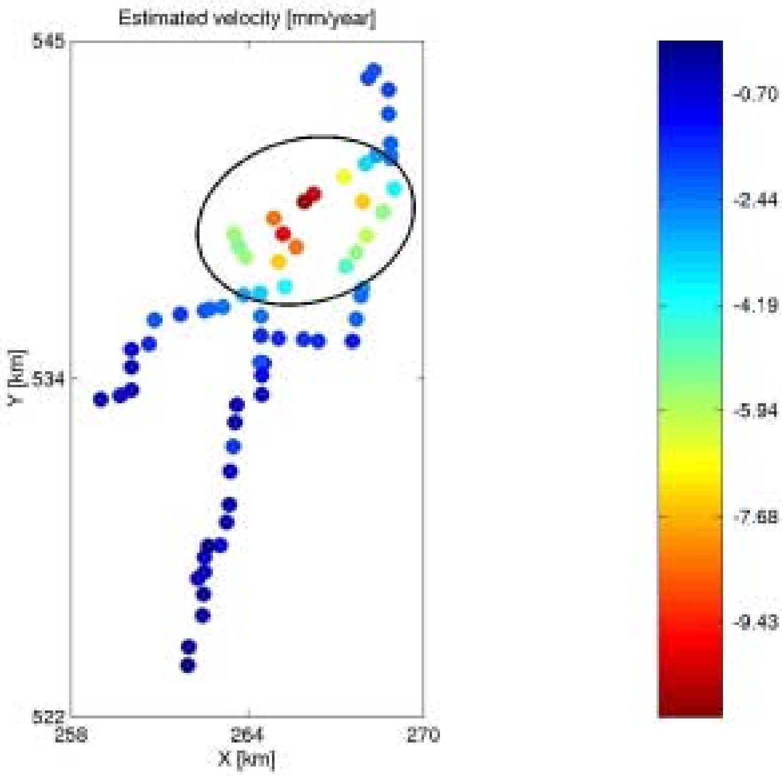

Figure 6: Estimated benchmark velocities of the Roswinkel data set.

7. Application of the approach to primary levellings

All available data constituting the primary levelling sets, i.e. the data from the second through fifth primary levelling campaign, were subjected to the software. Although the previous example in section 6 is a successful one, thereby proving the quality of the program, this important data set still faces unsolved problems. Consequently, no results have, as yet, been derived. Apart from earlier mentioned problems concerning reduction, the rounding of dates is not at all straightforward. If for instance one half of a network trajectory is measured in December and the other half in January, a rounding by year destroys this trajectory, thereby deleting vital information for a kinematic modelling of benchmarks. The problem is also largely due to the size and complexity of the data set. Only after clearing up all questions about the content of the data (what types of problems occur, and how often?), we can expect a successful and total run of the software, providing us with the estimated velocities of the benchmarks. One option would be to apply loose constraints to the estimated velocities, as suggested in section 4. This enables estimation, but complicates the interpretation and quality description of the estimated velocities. Also, a futile attempt was made to select a computable (regional) subset of the data. Although one would like to investigate all possibilities mentioned in section 4, it is advisable to preliminary adapt the software in order to handle the complexity of the data set.

8. Concluding remarks

Future research will be directed towards the problem the software faces: the complexity and size of the data. Further screening of the data set beforehand is also necessary in order to know which problems to tackle in the software. Next, the rounding of dates must be looked at.
Once benchmark velocities can be computed and visualized, a spatial-temporal extension of the model could supply an improved description of regional coherent vertical movements. So will a non-linear approach where e.g. a discontinuity in the benchmarks behavior is introduced. Finally, including geology as an external data source in the modelling will give extra insight in the vertical deformation of the Pleistocene sands.

The Geomatics Department of the Nederlandse Aardolie Maatschappij (NAM) is greatly acknowledged for placing the Roswinkel data at our disposal.

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