IDENTIFICATION OF PHYSICAL PARAMETERS BY MEANS OF DIFFERENTIAL EQUATIONS IN THE ADAPTIVE DYNAMIC FILTER MODEL

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Abstract

The description of the transfer function of an object, relating to its external influences (i.e. temperature), is a fundamental task within the framework of deformation analysis. By the definition of deterministic forcing functions, the answering of this question allows the prediction of relevant object reactions. Thus, critical system states can be simulated. In case of quantifiable physical structures, the time domain transfer function can be formulated by differential equations ('white box model'). This dynamic modeling method enables the consideration of the system capability to store and deliver energy with a time delay. Consequently, its behaviour under dynamic loads respectively the transitions between static loads can be described.

In the context of this paper, second order differential equations are combined with observations to a dynamic Kalman filter. The transition matrix is derived by spectral analysis of the state-space notation. The adaptive extension of the state vector with a physical part enables to estimate a priori unknown material parameters. Thus, the adaptation of the system equations to the observations is performed by a parametric identification.

The practical realization is effected by the identification of oscillating systems. Using simulations, the estimation problems concerning in general non-observable physical parameters in the state vector (defect in configuration) are discussed. If the behaviour of these parameters is described by an 'identity model', the filtering is primarily influenced by the accompanying 'random walk process'. It is shown that the creation of convergent solutions is possible, whereby, for the identification phase, only some few oscillations are necessary.

1. Introduction

Within the framework of deformation models it is to distinguish between two essential approaches, the 'descriptive' and the 'causative models' (WELSCH and others, 2000). Using descriptive models, the object transfer function and external influences (correcting variables) such as temperature, loads, etc. are neglected. As a consequence and main disadvantage, these aspects are not considered in deformation analysis. This neglecting does not appear in case of causal deformation models. Especially in the 'dynamic model' it is taken care to the object's capability to absorb, store and deliver energy in a time delayed deformation movement. Applicating this model, the temporal behaviour of the object is described very closely to reality. In contrast to the 'static model', the analysis and discussion of the transition period between static states of equilibrium is possible (HEUNECKE, 1995). This consideration is, for example, necessary in the case of thermal deformations because of the object's very slow temperature equalization (see KUHLMANN, PELZER 1997).

The identification (in ISERMANN, 1988 it is the experimental determination) of causative relationships may ensue 'non - parametric' or 'parametric' (ISERMANN 1988, HEUNECKE 1995, WELSCH and others 2000). In the first case, the object model exists as a pure 'input-output model' ('grey-' respectively 'black box model'). The abstract formulation of the transmission behaviour is the main disadvantage of this description method. In the case of parametric identification the physical object structure is totally described by differential equations. The parameters of these equations (i.e. rigidity, thermal conductivity etc.) can be physically interpreted. Its identification by geodetic measurements enables the control and discussion within the framework of deformation analysis. In the following sections such a 'white box model' in a dynamic representation is derived and discussed.

2. Integration of linear differential equations into the system equations of a Kalman filter

2.1 Fundamentals

The identification of a white box model requires the combination of a theoretically quantified structure model (set of differential equations) with object measurements discrete in locus and / or in time. For this task the Kalman filter is a particularly suitable tool. The structure model of the examined object is a part of its system equations. The experimental components are represented by the measuring equations. Using the innovation (discrepancy between measurements and model prediction) system and measuring equations are combined to an optimal estimation of the system state. The statistical evaluation of the innovation, in the sense of a test of consistency, allows the online control of the object behaviour. If there exist a priori insufficient known physical parameters in the system equations, they can be estimated and consequently identified by means of an adaptive filter extension (*section 2.3*).

In the following considerations it is assumed that the structure model only consists of time dependent ordinary differential equations. For this case the fundamental equation of linear dynamic elasticity is a typical example (MÖHLENBRINK/KRZYSTEK 1984, HEUNECKE 1995), whereby a building is abstracted as a system of masses, springs and dampers. The isolation of single elements of this vectorial differential equation leads to linear differential equations.

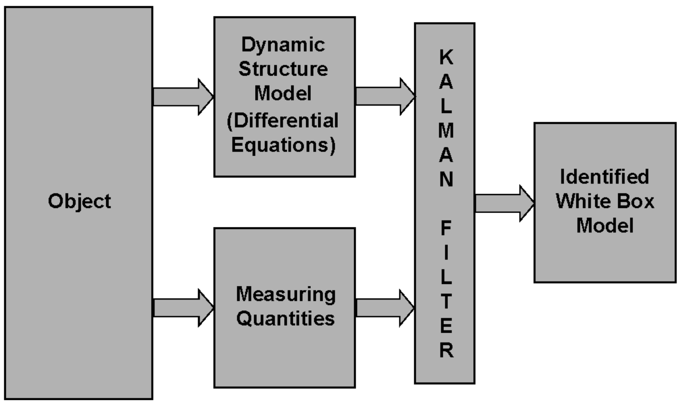

Figure 1: Identification of a white box model

In this paper the modelling of the object structure is limited to the domain of linear differential equations. In consideration of the finite sampling rate of geodetic observations the Kalman filter itself is represented discretely in time (GELB 1974, SCHRICK 1977, HEUNECKE 1995).

In SCHRICK (1977) the integration of a linear differential equation (LDE) in the system equations of a discrete Kalman filter is formally realized using the state-space notation. The scalar LDE of order n

$$y^{(n)} + a_{n-1}\, y^{(n-1)} + \cdots + a_0\, y \;=\; u + w \qquad (n > 0) \tag{1}$$

is a mathematical (part of a) model for a linear respectively linearized dynamic system. The variable y and its higher derivatives are the interesting scalar output quantities, u represents a scalar deterministic system input (correcting variable) and w a scalar stochastic input (disturbance variable) which is introduced because of an incomplete system quantification. Defining a number of n state parameters

$$x_1 = y\,,\, x_2 = \dot{y}\,,\; \ldots\;, x_n = y^{(n-1)} \tag{2}$$

at which n is corresponding with the order of the LDE, (1) is transformed in a vectorial differential equation consisting of n first order LDE's, the continuous state-space model (SCHRICK, 1977).

$$\dot{\boldsymbol{x}} = \boldsymbol{F}\,\boldsymbol{x} + \boldsymbol{G}\,u + \boldsymbol{D}\,w \qquad \text{with} \quad \boldsymbol{x}^{\mathrm{T}} = (x_1, x_2, \ \dots \ , x_n)^{\mathrm{T}} \tag{3}$$

In (3) $\boldsymbol{x}$ is the state vector, $\boldsymbol{F}$ the square system dynamics matrix, $\boldsymbol{G}$ the coefficient matrix of continuous correcting variable and $\boldsymbol{D}$ the coefficient matrix of continuous disturbance variable.

The transition to a discrete representation, which corresponds with the system equations of the Kalman filter, is realized using the following relationship (GELB 1974, SCHRICK 1977) :

$$\boldsymbol{x}(t_{k+1}) = \boldsymbol{\Phi}(t_{k+1}, t_k)\,\boldsymbol{x}(t_k) + \int_{t_k}^{t_{k+1}} \boldsymbol{\Phi}(t_{k+1}, \tau)\,\boldsymbol{G}\,u(\tau)\,d\tau + \int_{t_k}^{t_{k+1}} \boldsymbol{\Phi}(t_{k+1}, \tau)\,\boldsymbol{D}\,w(\tau)\,d\tau \tag{4}$$

$\boldsymbol{\Phi}$ is the transition matrix. Defining the requirements that $u(t) \approx u(t_k)$ and $w(t) \approx w(t_k)$ in $[t_k, t_{k+1}]$ (4) can be transformed to the well known representation

$$\begin{aligned} &\boldsymbol{x}(t_{k+1}) = \boldsymbol{T}(t_{k+1}, t_k)\ \boldsymbol{x}(t_k) + \boldsymbol{B}(t_{k+1}, t_k)\ u(t_k) + \boldsymbol{S}(t_{k+1}, t_k)\ w(t_k) \qquad \text{with} \\ &\boldsymbol{T}(t_{k+1}, t_k) = \boldsymbol{\Phi}(t_{k+1}, t_k) \ \ ; \ \boldsymbol{B}(t_{k+1}, t_k) = \int_{t_k}^{t_{k+1}} \boldsymbol{\Phi}(t_{k+1}, \tau)\,\boldsymbol{G}\,d\tau \ \ ; \ \boldsymbol{S}(t_{k+1}, t_k) = \int_{t_k}^{t_{k+1}} \boldsymbol{\Phi}(t_{k+1}, \tau)\,\boldsymbol{D}\ d\tau \end{aligned} \tag{5}$$

with $\boldsymbol{T}$ = transition matrix (= $\boldsymbol{\Phi}$), $\boldsymbol{B}$ = coefficient matrix of discrete correcting variable and $\boldsymbol{S}$ = coefficient matrix of discrete disturbance variable. Equation (5) is the piecewise solution of LDE (1). The state vextor $\boldsymbol{x}(t_k)$ consists of the initial values of the differential equation at time t_k, which are used to update its solution.

2.2 Determination of the transition matrix *T* by means of equivalent polynomials

The determination of the transition matrix $\boldsymbol{T}$ can be realized by different strategies. In case of a time invariant or assumed as piecewise time invariant square system dynamics matrix $\boldsymbol{F}$ the transition matrix is derived by a matrix exponential (GELB, 1974).

$$\boldsymbol{T}(t_{k+1}, t_k) = e^{\boldsymbol{F}\,(t_{k+1} - t_k)} = e^{\boldsymbol{F}\,\Delta t} \qquad ; \qquad \Delta t = t_{k+1} - t_k \tag{6}$$

Usually this calculation is effected numerically cutting off the power series representation of (6) (GELB 1974, SCHRICK 1977, HEUNECKE 1995). Considering the adaptive Kalman filter extension in *section 2.4* a strict analytic formulation is desired, avoiding the discussion of errors of approximation and creating a compact solution suitable for further analytic examinations. HUEP (1986) realizes the analytic solution by 'Laplace transformations'. In this paper an approach corresponding with in geodesy well known spectral analysis of matrices is presented.

The basic idea of the following approach is substituting the matrix exponential in (6)

$$\mathrm{f}(\boldsymbol{F}) = e^{\boldsymbol{F}\,\Delta t} \quad \left(= \boldsymbol{T}(t_{k+1}, t_k)\right) \tag{7}$$

by a polynomial $\varphi(\boldsymbol{F})$, which is equivalent to the function $\mathrm{f}(\boldsymbol{F})$. In comparison with the representation of (7) by an infinite power series this polynomial has a finite degree. Consequently a complete analytical formulation is possible.

$$\mathrm{f}(\boldsymbol{F}) = e^{\boldsymbol{F}\,\Delta t} = \sum_{k=0}^{\infty} \frac{\boldsymbol{F}^k}{k!} \Delta t \overset{!}{=} \varphi(\boldsymbol{F}) \tag{8}$$

In *figure 2* the practical determination of the equivalent polynomial φ belonging to the matrix function $\mathrm{f}(\boldsymbol{F})$ in (8) and the derivation

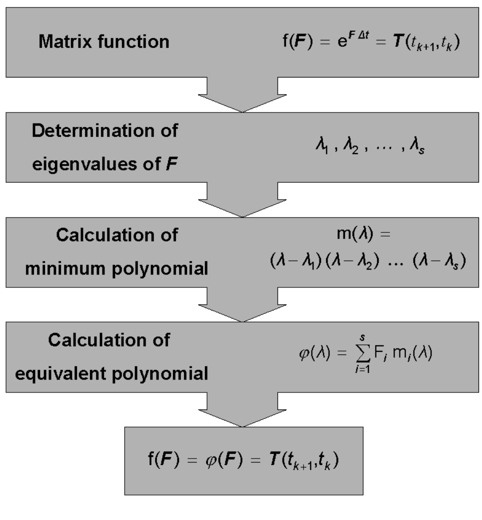

Figure 2: Determination of transition matrix ***T***

of the transition matrix $\boldsymbol{T}$ are shown. The mathematical terms and the special requirements for the system dynamics matrix $\boldsymbol{F}$ are motivied in the following considerations. Having determined the transition matrix $\boldsymbol{T}$ the matrices $\boldsymbol{B}$ and $\boldsymbol{S}$ can be calculated using the definite integrals in (5). Consequently the system equations of the discrete Kalman filter are defined.

An important theoretical fundamental can be found in the 'Cayley-Hamilton theorem'. In this theorem it is pointed out that any square matrix $\boldsymbol{A}$ (n, n) is always a 'zero' of its own characteristic polynomial $\mathrm{p}(\lambda)$. This polynomial is derived from the eigenvalue problem (EVP) belonging to $\boldsymbol{A}$. Consequently $\boldsymbol{A}$ fulfills the following equation (9). The further considerations are based on ZURMÜHL (1964).

$$\begin{aligned} \mathrm{p}(\lambda) &= \det(\boldsymbol{A}-\lambda\boldsymbol{E}) = \lambda^n + a_{n-1}\lambda^{n-1} + \ldots + a_1\lambda + a_0 \qquad \text{EVP} \\ \mathrm{p}(\boldsymbol{A}) &= \boldsymbol{A}^n + a_{n-1}\boldsymbol{A}^{n-1} + \ldots + a_1\boldsymbol{A} + a_0\boldsymbol{E} \overset{!}{=} \boldsymbol{O} \end{aligned} \tag{9}$$

At first the discussion of a matrix function $\mathrm{f}(\boldsymbol{A})$ is restricted to the special case of a matrix polynomial $\mathrm{P}(\boldsymbol{A})$. Utilizing (9) the equivalent polynomial $\varphi(\boldsymbol{A})$ can be calculated by simple polynomial factorization with the corresponding scalar polynomials $\mathrm{P}(\lambda)$ respectively $\mathrm{p}(\lambda)$:

$$\frac{\mathrm{P}(\lambda)}{\mathrm{p}(\lambda)} = \mathrm{g}^*(\lambda) \quad ; \quad \text{remainder } \varphi^*(\lambda)$$

$$\Rightarrow \; \mathrm{P}(\lambda) = \mathrm{p}(\lambda)\,\mathrm{g}^*(\lambda) + \varphi^*(\lambda) \tag{10}$$

$$\Rightarrow \; \mathrm{P}(\boldsymbol{A}) = \underbrace{\mathrm{p}(\boldsymbol{A})}_{\boldsymbol{O}}\,\mathrm{g}^*(\boldsymbol{A}) + \varphi^*(\boldsymbol{A}) \qquad \Rightarrow \; \mathrm{P}(\boldsymbol{A}) = \varphi^*(\boldsymbol{A})$$

Executing the factorization in (10) a matrix polynomial $\mathrm{P}(\boldsymbol{A})$ of degree $q \geq n$ can be represented by an equivalent polynomial $\varphi^*(\boldsymbol{A})$ of degree $r \leq n\text{-}1$. The degree r of the equivalent polynomial can further be reduced substituting the divisor p by the minimum polynomial m. The minimum polynomial is the polynomial of lowest degree fulfilling $\mathrm{m}(\boldsymbol{A}) = \boldsymbol{O}$ (ZURMÜHL, 1964). Consequently (10) can be substituted by :

$$\mathrm{P}(\boldsymbol{A}) = \underbrace{\mathrm{m}(\boldsymbol{A})}_{\boldsymbol{O}}\mathrm{g}(\boldsymbol{A}) + \varphi(\boldsymbol{A}) = \varphi(\boldsymbol{A}) \tag{11}$$

At this point the construction of m is only introduced for the very important class of diagonalizable matrices. Diagonalizable square matrices are of dimension n and own exactly n linearly independent eigenvectors. For this case a matrix $\boldsymbol{A}$ with s different eigenvalues λ_i owns a minimum polynomial m of degree s (ZURMÜHL, 1964) :

$$\mathrm{m}(\lambda) = (\lambda-\lambda_1)(\lambda-\lambda_2)\ldots(\lambda-\lambda_s) \qquad \text{with } \mathrm{m}(\boldsymbol{A}) = \boldsymbol{O} \tag{12}$$

The power series in (8) is a matrix polynomial of infinite degree. Using a minimum polynomial it can, in principle, be reduced to a finite equivalent polynomial. Actually the practical realization of this idea cannot be directly done by polynomial factorization. In ZURMÜHL (1964) an approach is described which can be applied to matrix polynomials of infinite degree and consequently to matrix functions with power series representation.

Presuming a matrix function $\mathrm{f}(\boldsymbol{A})$ with diagonalizable matrix $\boldsymbol{A}$ the utilization of its minimum polynomial m of degree s (12) leads to the following 'reduction' :

$$\begin{aligned} &\varphi(\lambda) = \sum_{i=1}^{s} \mathrm{F}_i\,\mathrm{m}_i(\lambda) \qquad \text{(equivalent polynomial of degree } s-1) \\ &\text{with} \quad \mathrm{m}_i(\lambda) = \frac{\mathrm{m}(\lambda)}{(\lambda - \lambda_i)} \qquad \text{and} \qquad \mathrm{F}_i = \mathrm{F}_i(\lambda_i) = \frac{\mathrm{f}(\lambda_i)}{\mathrm{m}_i(\lambda_i)} \end{aligned} \tag{13}$$

In (13) the equivalent polynomial $\varphi(\lambda)$ is constructed as a sum of s 'Lagrange' polynomials $m_i(\lambda)$ which are combined with coefficients F_i including the eigenvalues $f(\lambda_i)$ of $f(\boldsymbol{A}.)$.

Matrix functions with non-diagonalizable matrices can also be substituted by equivalent polynomials (ZURMÜHL, 1964), but the representation is more complicated. Considering the practical application in *section 2.3* the system dynamics matrix $\boldsymbol{F}$ is diagonalizable. The case of non-diagonalizability is not further discussed in this paper.

With (12) and (13) the sequence shown in *figure 2* is completely described. In the following section the calculation is illustrated by an example. This example is selected from a special and important class of dynamic systems: the mass-spring-damper-systems.

2.3 The system equations of the mass-spring-damper-system

Within the domain of structure models the fundamental equation of linear dynamic elasticity represents a very important element in context with the physical interpretable modelling of the object's transmission behaviour (*section 2.1*). In the following the one-dimensional movement of an object point is described. This corresponds with its movement in the direction of only one axis of the object coordinate system neglecting algebraic couplings with other coordinates. In this case the fundamental equation is reduced to a linear differential equation of second order which represents an one-dimensional dynamical system.

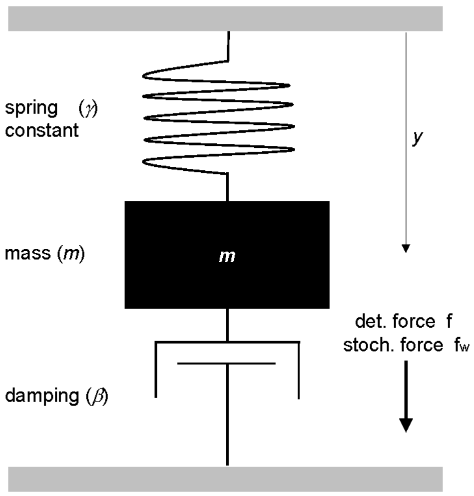

Figure 3: Mass-spring-damper-system

$$m\,\ddot{y} + \beta\,\dot{y} + \gamma\,y \;=\; f + f_{\mathrm{w}} \tag{14}$$

With (14) a mass-spring-damper-system is described at which m is the mass, β the damping and γ the spring constant (see also *figure 3*). On the right side of the LDE f represents an external deterministic force (correcting variable) and f_{w} a stochastic force (disturbance variable).

Dividing (14) by m the equation is transformed into a representation corresponding with LDE (1). Introducing the substitutions ω_0 and ξ normally used in mechanics equation (14) can be converted into the linear differential equation (15). In this equation the quantity ω_0 is the natural angular frequency, that means the angular frequency of the autonomous continous oscillation. ξ is a damping factor specifying the oscillatory behaviour of the system ($\xi < 1$ oscillation, $\xi = 1$ aperiodic limit case, $\xi > 1$ creeping). The following representation is conforming to SCHRICK (1977) and is the fundamental equation of all further considerations :

$$\ddot{y} + 2\xi\,\omega_0\,\dot{y} + \omega_0^2\,y \;=\; u + w \qquad \text{mit} \quad \omega_0 \;=\; \sqrt{\frac{\gamma}{m}} \quad ; \quad \xi \;=\; \frac{\beta}{2\,\omega_0\,m} \tag{15}$$

The right side of LDE (15) consists of accelerations u as correcting and w as disturbance variable. Now the transition to the state-space notation is the first step to integrate (15) into the system equations of a Kalman filter.

$$\underbrace{\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix}}_{\dot{\boldsymbol{x}}} = \underbrace{\begin{pmatrix}0 & 1\\ -\omega_0^2 & -2\xi\omega_0\end{pmatrix}}_{\boldsymbol{F}} \underbrace{\begin{pmatrix}x_1\\ x_2\end{pmatrix}}_{\boldsymbol{x}} + \underbrace{\begin{pmatrix}0\\ 1\end{pmatrix}}_{\boldsymbol{G}} u + \underbrace{\begin{pmatrix}0\\ 1\end{pmatrix}}_{\boldsymbol{D}} w \qquad ; \qquad \begin{aligned} x_1 &= y\\ x_2 &= \dot{y}\end{aligned} \tag{16}$$

The transition matrix $\boldsymbol{T}$ is determined with the method described in *section 2.2*. The eigenvalues of the system dynamics matrix $\boldsymbol{F}$ are :

$$\det(\boldsymbol{F} - \lambda \boldsymbol{E}) = 0 \;\Rightarrow\; \lambda_{1,2} = -\xi\omega_0 \pm \omega_0\sqrt{\xi^2 - 1} \tag{17}$$

In (17) it is shown that all eigenvalues are different, so that the diagonalizability of $\boldsymbol{F}$ immediately can be concluded (ZURMÜHL, 1964). In the following with $\xi < 1$ an oscillating system is considered. Consequently the eigenvalues become complex.

$$\lambda_{1,2} = \underbrace{-\xi\omega_0}_{\sigma} \pm \mathrm{j}\underbrace{\omega_0\sqrt{1\text{-}\xi^2}}_{\omega} = \sigma \pm \mathrm{j}\omega \qquad \text{with } \mathrm{j} = \text{imaginary unit} \tag{18}$$

The minimum polynomial m(λ) is derived with (12) :

$$\mathrm{m}(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2) = (\lambda - \sigma - \mathrm{j}\omega)\,(\lambda - \sigma + \mathrm{j}\omega) \tag{19}$$

Using (7) and (13) the equivalent polynomial φ(λ) of the matrix exponential can be determined.

$$\varphi(\lambda) = \mathrm{e}^{\sigma\Delta t}\left[\frac{1}{\omega}\sin(\omega\Delta t)\lambda + \cos(\omega\Delta t) - \frac{\sigma}{\omega}\,sin(\,\omega\Delta t\,)\right] \tag{20}$$

The transition matrix $\boldsymbol{T}$ of the mass-spring-damper-system follows as :

$$\begin{aligned}\boldsymbol{T}(t_{k+1},t_k) &= \varphi(\boldsymbol{F}) = \mathrm{e}^{\sigma\Delta t}\left[\frac{1}{\omega}\sin(\omega\Delta t)\boldsymbol{F} + \left(\cos(\omega\Delta t) - \frac{\sigma}{\omega}\,sin(\omega\Delta t)\right)\boldsymbol{E}\right] \\ &= \mathrm{e}^{\sigma\Delta t}\begin{pmatrix} \cos(\omega\Delta t) - \frac{\sigma}{\omega}\sin(\omega\Delta t) & \frac{1}{\omega}\sin(\omega\Delta t) \\ -\frac{\omega_0^2}{\omega}\sin(\omega\Delta t) & \frac{\sigma}{\omega}\sin(\omega\Delta t) + \cos(\omega\Delta t)\end{pmatrix}\end{aligned} \tag{21}$$

Inserting $\boldsymbol{G}$ and $\boldsymbol{D}$ from (16) and $\boldsymbol{T}$ from (21) in the definite integrals (5) the coefficient matrices of discrete correcting variable $\boldsymbol{B}$ and of discrete disturbance variable $\boldsymbol{S}$ of the Kalman filter are calculated. As $\boldsymbol{G} \equiv \boldsymbol{D}$ the result is :

$$\boldsymbol{B}(t_{k+1},t_k) = \boldsymbol{S}(t_{k+1},t_k) = \begin{pmatrix} \frac{1}{\sigma^2 + \omega^2}\left[1 + \mathrm{e}^{\sigma\Delta t}\left(\frac{\sigma}{\omega}\sin(\omega\Delta t) + \cos(\omega\Delta t)\right)\right] \\ \frac{1}{\omega}\mathrm{e}^{\sigma\Delta t}\sin(\omega\Delta t)\end{pmatrix} \tag{22}$$

With (21) and (22) the mass-spring-damper-system is integrated into the system equations of the Kalman filter. As shown in (16) the state vector $\boldsymbol{x}$ for the present case only consists of locus and velocity of mass m.

2.4 Extension of the system equations to the adaptive Kalman filter

The comparison of the coefficients between the dynamic structure model (15) and LDE (1) results in the following representation which is equivalent to (15) :

$$\ddot{y} + a_1\dot{y} + a_0 y = u + w \quad \text{with } a_0 = \omega_0^2 = \frac{\gamma}{m} \text{ and } a_1 = 2\xi\,\omega_0 = \frac{\beta}{m} \tag{23}$$

Model (23) is containing the physically interpretable parameters a_0 and a_1 which have to be adapted to the empirical measurements. This adaptation is the main task of the parametrical identification and requires the extension of the 'descriptive' state vector $\boldsymbol{x}_\mathrm{d}$ in (16) with an adaptive 'physical' part $\boldsymbol{x}_\mathrm{p}$.

$$\boldsymbol{x}^{\mathrm{T}} = (\boldsymbol{x}_{\mathrm{d}} \;\; \boldsymbol{x}_{\mathrm{p}})^{\mathrm{T}} = (x_1 \;\; x_2 \mid a_0 \;\; a_1)^{\mathrm{T}} = \left(y \;\; \dot{y} \mid \gamma/m \;\; \beta/m\right)^{\mathrm{T}} \tag{24}$$

In the system equations of the Kalman filter the update of $\boldsymbol{x}_{\mathrm{p}}$ is realized by a 'random walk process' with disturbance variables $\boldsymbol{w}_{\mathrm{p}}$. Using (5), (21), (22) and (24) the extended system equations are defined as

$$\begin{aligned}
\underset{2,1}{\underline{\tilde{\boldsymbol{x}}_{\mathrm{d}}(t_{k+1})}} &= \underset{2,2}{\underline{\boldsymbol{T}(t_{k+1},t_k)}}\,\underset{2,1}{\underline{\tilde{\boldsymbol{x}}_{\mathrm{d}}(t_k)}} + \underset{2,1}{\underline{\boldsymbol{B}(t_{k+1},t_k)}}\,\underset{1,1}{\underline{\tilde{u}(t_k)}} + \underset{2,1}{\underline{\boldsymbol{S}(t_{k+1},t_k)}}\,\underset{1,1}{\underline{w(t_k)}} &&= \underset{2,1}{\underline{\Psi_{\mathrm{d}}}}(\tilde{\boldsymbol{x}}_{\mathrm{d},k},\tilde{\boldsymbol{x}}_{\mathrm{p},k},\tilde{u}_k,w_k)\\
\underset{2,1}{\underline{\tilde{\boldsymbol{x}}_{\mathrm{p}}(t_{k+1})}} &= \underset{2,1}{\underline{\tilde{\boldsymbol{x}}_{\mathrm{p}}(t_k)}} + \underset{2,1}{\underline{\boldsymbol{w}_{\mathrm{p}}(t_k)}} &&= \underset{2,1}{\underline{\Psi_{\mathrm{p}}}}(\tilde{\boldsymbol{x}}_{\mathrm{p},k},\boldsymbol{w}_{\mathrm{p},k})
\end{aligned} \tag{25}$$

at which in the following ~ describes the true state quantities $\boldsymbol{x}$ respectively acceleration u.

Assuming $\mathrm{E}\{w_k\} = 0$ and $\mathrm{E}\{\boldsymbol{w}_{\mathrm{p},k}\} = \boldsymbol{o}$ the prediction $\bar{\boldsymbol{x}}_{k+1}$ is calculated with the deterministic part of (25) inserting the estimated state vector $\hat{\boldsymbol{x}}_k$ and the measured acceleration u_k. Because of non-linearity of the physical state parameters $\boldsymbol{x}_{\mathrm{p}}$ in $\boldsymbol{T}$, $\boldsymbol{B}$ and $\boldsymbol{S}$ the determination of the covariance matrix of the prediction requires the linearization of (25). The linearized prediction errors are :

$$\begin{aligned}
\tilde{\boldsymbol{x}}_{\mathrm{d},k+1} - \bar{\boldsymbol{x}}_{\mathrm{d},k+1} &= \frac{\partial\Psi_{\mathrm{d}}}{\partial\boldsymbol{x}_{\mathrm{d},k}}(\tilde{\boldsymbol{x}}_{\mathrm{d},k} - \hat{\boldsymbol{x}}_{\mathrm{d},k}) + \frac{\partial\Psi_{\mathrm{d}}}{\partial\boldsymbol{x}_{\mathrm{p},k}}(\tilde{\boldsymbol{x}}_{\mathrm{p},k} - \hat{\boldsymbol{x}}_{\mathrm{p},k}) + \frac{\partial\Psi_{\mathrm{d}}}{\partial u_k}(\tilde{u}_k - u_k) + \frac{\partial\Psi_{\mathrm{d}}}{\partial w_k}w_k\\
\tilde{\boldsymbol{x}}_{\mathrm{p},k+1} - \bar{\boldsymbol{x}}_{\mathrm{p},k+1} &= \frac{\partial\Psi_{\mathrm{p}}}{\partial\boldsymbol{x}_{\mathrm{p},k}}(\tilde{\boldsymbol{x}}_{\mathrm{p},k} - \hat{\boldsymbol{x}}_{\mathrm{p},k}) + \frac{\partial\Psi_{\mathrm{p}}}{\partial\boldsymbol{w}_{\mathrm{p},k}}\boldsymbol{w}_{\mathrm{p},k}
\end{aligned} \tag{26}$$

Equation (26) leads to the following matrix representation :

$$\begin{pmatrix} \tilde{\boldsymbol{x}}_{\mathrm{d},k+1} - \bar{\boldsymbol{x}}_{\mathrm{d},k+1} \\ \tilde{\boldsymbol{x}}_{\mathrm{p},k+1} - \bar{\boldsymbol{x}}_{\mathrm{p},k+1} \end{pmatrix} = \begin{pmatrix} \underset{2,2}{\underline{\boldsymbol{T}_{k+1,k}}} & \underset{2,2}{\underline{\boldsymbol{T}_{\mathrm{p},k+1,k}}} \\ \boldsymbol{O} & \underset{2,2}{\underline{\boldsymbol{E}}} \end{pmatrix} \begin{pmatrix} \tilde{\boldsymbol{x}}_{\mathrm{d},k} - \hat{\boldsymbol{x}}_{\mathrm{d},k} \\ \tilde{\boldsymbol{x}}_{\mathrm{p},k} - \hat{\boldsymbol{x}}_{\mathrm{p},k} \end{pmatrix} + \begin{pmatrix} \underset{2,1}{\underline{\boldsymbol{B}_{k+1,k}}} \\ \boldsymbol{O} \end{pmatrix} (\tilde{u}_k - u_k) + \begin{pmatrix} \underset{2,1}{\underline{\boldsymbol{S}_{k+1,k}}} & \boldsymbol{O} \\ \boldsymbol{O} & \underset{2,2}{\underline{\boldsymbol{E}}} \end{pmatrix} \begin{pmatrix} w_k \\ \boldsymbol{w}_{\mathrm{p},k} \end{pmatrix} \tag{27}$$

with matrices $\boldsymbol{T}_{k+1,k}$, $\boldsymbol{B}_{k+1,k}$ and $\boldsymbol{S}_{k+1,k}$ known from (25). The matrix $\boldsymbol{T}_{\mathrm{p},k+1,k}$ is the Jacobi matrix derived from the descriptive part ψ_{d} of the system equations :

$$\underset{2,2}{\underline{\boldsymbol{T}_{\mathrm{p},k+1,k}}} = \frac{\partial\psi_{\mathrm{d}}}{\partial\boldsymbol{x}_{\mathrm{p},k}} = \begin{pmatrix} \dfrac{\partial\psi_{\mathrm{d}}}{\partial a_0} & \dfrac{\partial\psi_{\mathrm{d}}}{\partial a_1} \end{pmatrix} \tag{28}$$

Using (27) for the propagation of errors the crosscovariances between the estimated state vector $\hat{\boldsymbol{x}}_k = (\hat{\boldsymbol{x}}_{\mathrm{d},k} \;\; \hat{\boldsymbol{x}}_{\mathrm{p},k})^{\mathrm{T}}$, the measured acceleration u_k, and the disturbance variables w_k (stochastic acceleration) and $\boldsymbol{w}_{\mathrm{p},k}$ (of the physical part of the state vector) are neglected. $u - \tilde{u}$, w and $\boldsymbol{w}_{\mathrm{p}}$ are further assumed to be 'white noise' (no temporary correlations). Consequently the stochastic model of the linearized system equations (27) is defined as :

$$\boldsymbol{\Sigma}_{l^*l^*,k} = \begin{pmatrix} \underset{4,4}{\underline{\boldsymbol{\Sigma}_{\hat{x}\hat{x},k}}} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \underset{1,1}{\underline{\boldsymbol{\Sigma}_{uu,k}}} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & \underset{3,3}{\underline{\boldsymbol{\Sigma}_{ww,k}}} \end{pmatrix} \qquad \text{mit} \;\; \boldsymbol{\Sigma}_{ww,k} = \begin{pmatrix} \underset{1,1}{\underline{\boldsymbol{\Sigma}_{ww,k}}} & \boldsymbol{O} \\ \boldsymbol{O} & \underset{2,2}{\underline{\boldsymbol{\Sigma}_{w_{\mathrm{p}}w_{\mathrm{p}},k}}} \end{pmatrix} \tag{29}$$

Because of the high sampling rate of geodetic measuring sensors, temporary correlations between the measuring quantities of acceleration u must be supposed. Furtheron the disturbance influences are also temporary correlated. The consideration of these correlations in a 'form filter' (HUEP, 1986) is not discussed in this article.

3. Identification of a mass-spring-damper-system

For the identification of a mass-spring-damper-system simulated observations are generated. Using the exact solution of LDE (23) and defining the physical parameters as

$$a_0 = 40\,\mathrm{s}^{-2} \quad ; \quad a_1 = 1\,\mathrm{s}^{-1} \tag{30}$$

and the harmonic acceleration (correcting variable) as

$$u(t) = u_0 + u_1 \sin(\omega t + \varphi) \qquad u_0 = 9.81\,\mathrm{ms}^{-2};\ u_1 = 1.0\,\mathrm{ms}^{-2};\ \omega = \pi\ \mathrm{Hz}\ ;\ \varphi = 0 \tag{31}$$

a reference oscillation $y(t)$ starting from its steady position is calculated with $\omega_0 \approx 2\pi$ Hz and an impressed frequency (after response time) of $\nu = 0.5$ Hz. Assuming suitable sensors the observations y of locus of mass m and of the influencing acceleration u are created by sampling and adding noise to the reference trajectories with $\Delta t = 0.05$ s, $\sigma_y = 2$ mm and $\sigma_u = 5$ mm s^{-2}. The sampling rate of 20 Hz corresponds with about 20 samples per oscillation (during response time). With (30) the reference quantities of the identification task are defined. In *figure 4* the observations of locus y and acceleration u are shown for a period of 10 s.

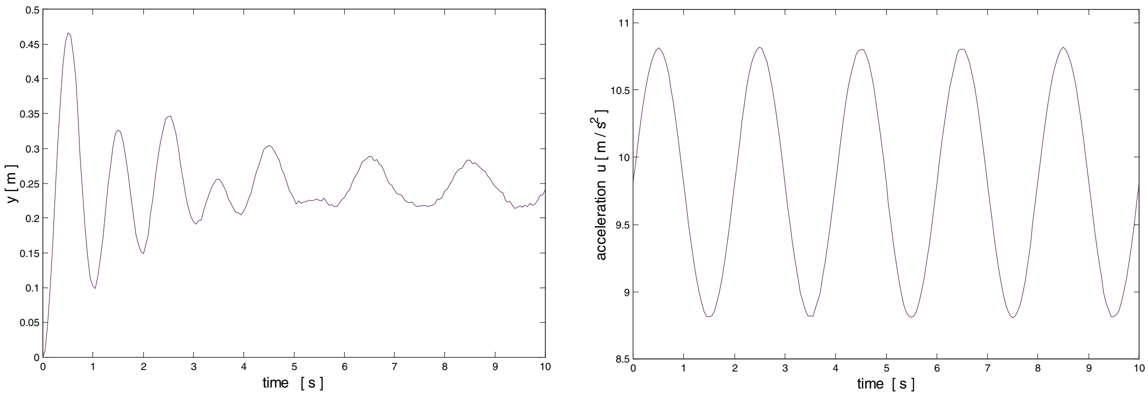

Figure 4: Simulated observations of locus y and acceleration u

Now, we assume that the physical parameters a_0 and a_1 of the dynamic structure model (23) are a priori insufficiently known. It is the task of the parametric identification to identify these quantities with the help of the extended system equations derived in *section 2.4* and the observations y and u.

As initial values of the physical part $\boldsymbol{x}_\mathrm{p}$ of the state vector the following quantities are chosen :

$$\boldsymbol{x}_{\mathrm{p},0}: \quad a_{0,0} = 50\,\mathrm{s}^{-2} \quad ; \quad a_{1,0} = 1.5\,\mathrm{s}^{-1} \tag{32}$$

At epoch k the measuring equation of the Kalman filter is defined as :

$$y_\mathrm{k} + v_\mathrm{k} = \boldsymbol{A}\,\hat{\boldsymbol{x}}_\mathrm{k} = (1\ \ 0\ \ 0\ \ 0)\,(\hat{x}_{1,\mathrm{k}}\ \ \hat{x}_{2,\mathrm{k}}\,|\,\hat{a}_{0,\mathrm{k}}\ \ \hat{a}_{1,\mathrm{k}})^\mathrm{T} \tag{33}$$

The composition of stochastic model (29) and covariance matrix of measurements is realized time-invariantly with realistic assumptions. It is very important to remark that the covariance matrix $\boldsymbol{\Sigma}_{w\mathrm{p}w\mathrm{p},k}$ of the random walk process is equated to zero. A deviation of zero immediately creates a more instable parameter estimation of a_0 and a_1 and in the worst case a filter divergence. Detailed examinations concerning the influence of $\boldsymbol{w}_\mathrm{p}$ have yet to come.

At epoch k the matrices of the system equations are composed using $\hat{\boldsymbol{x}}_{k-1}$. In *figures 5* and *6* the progress of the filter is shown for the estimations of a_0 and a_1 respectively their standard deviations in the period of 10 s after filter initialization with (32).

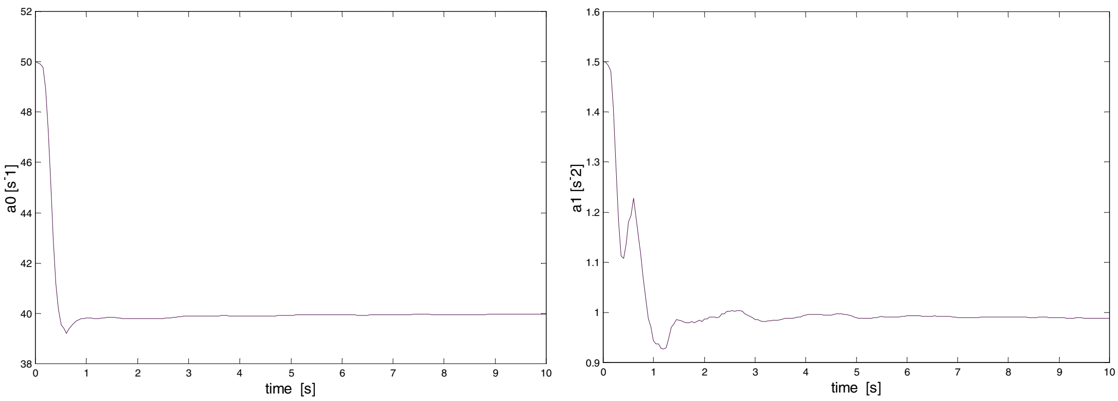

Figure 5: Progress of estimated a_0 and a_1

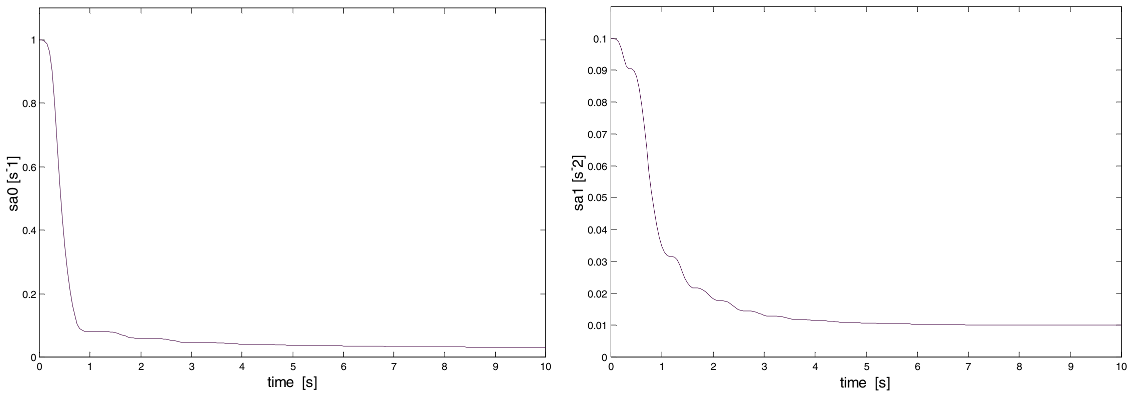

Figure 6: Progress of standard deviations s_{a0} and s_{a1}

In (33) it can be seen that only the locus x_1 is observable. All other quantities of $\boldsymbol{x}$ are non-observable. Nevertheless the filters capability to indentify the physical part $\boldsymbol{x}_\mathrm{p}$ is strongly influenced by matrix $\boldsymbol{T}_{\mathrm{p},k+1,k}$ (28) which produces algebraic correlations between descriptive and physical part of the state vector (HEUNECKE, 1995). The consequence of these correlations is the 'autonomous' reduction of the initial standard deviations in $\boldsymbol{x}_\mathrm{p}$. This may be used as an indicator concerning identifiability (SCHRICK 1977, HEUNECKE 1995).

In *figure 5* the estimations of $\boldsymbol{x}_\mathrm{p}$ converge in about 5 s. Their final level corresponds with the reference values in (30). In the same period the standard deviations of $\boldsymbol{x}_\mathrm{p}$ are also reduced to a constant low level (*figure 6*). This means an identification phase of 5 observed oscillations (*figure 4*). Consequently considering the estimations of a_0 and a_1, calculated by the adaptive Kalman filter, in this example the dynamic white box model (23) can be assumed as identified.

The small deviations between estimations and reference values (0.1 % in a_0 and 1 % in a_1, see *figure 5*) may be reduced introducing more suitable stochastic models and increasing the sampling rate. In general the estimation of parameter a_1 shows a more instable progress than a_0. Obviously the reason for this behaviour is that because of the forced oscillation, parameter a_1 (dependent on damping constant β) is worse identifiable than parameter a_0 (dependent on spring constant γ). The examination of autonomous oscillations (i.e. system impulse response) resul-

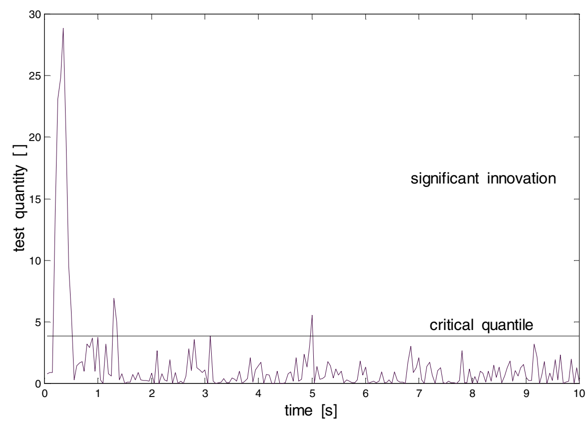

Figure 7: Test of innovation d

ted in more smooth estimations. Applying the normal test for innovation d (WELSCH and others, 2000) a result is received conforming to the previous considerations (*figure 7*). Nevertheless the test result is extremely dependent on the introduced stochastic quantities. Furthermore determining the degrees of freedom of the test quantity the autocorrelations of d are neglected. Thus the test can be assumed as too optimistic. To create a statistic indicator for the significance of innovation a modified test strategy should be used.

4. Conclusions and perspectives

In this paper the parametric identification of dynamic structure models is exemplarily realized using a simulated mass-spring-damper-system. The physically interpretable quantities γ/m and β/m are estimated. It is shown that convergent solutions can be created corresponding with the reference values. But it is also evident that the parameter identifiability is dependent on the nature of influencing correcting variables. In analogy to the non-parametric identification this fact has to be taken into account in practical applications. By means of the identified structure model (23) the simulation of the dynamic behaviour of mass m is possible considering (within physical restrictions) any correcting variables.

The experimental realization concerning the parametrical identification of a non-stationary temperature model which is based on parabolic differential equations is in work.

References

GELB, A. (1974): *Applied Optimal Estimation*. The M.I.T. Press, Cambridge, Massachusetts

HEUNECKE, O. (1995): *Zur Identifikation und Verifikation von Deformationsprozessen mittels adaptiver KALMAN-Filterung (Hannoversches Filter)*. Wissenschaftliche Arbeiten der Fachrichtung Vermessungswesen der Universität Hannover, Hannover, Nr. 208

HUEP, W. (1986): *Zur Positionsschätzung im gestörten KALMAN-Filter am Beispiel eines manövrierenden Wasserfahrzeuges.* Wissenschaftliche Arbeiten der Fachrichtung Vermessungswesen der Universität Hannover, Hannover, Nr. 143

ISERMANN, R. (1988): *Identifikation dynamischer Systeme*. Band 1, Springer Verlag, Berlin

KUHLMANN, H. / PELZER, H. (1997): *Models of Deformation Analysis*. Altan M. / Gründig L.: Second Turkish-German Joint Geodetic Days, Berlin

MÖHLENBRINK, W. / KRZYSTEK, P. (1984): *Messung windinduzierter Bewegungen an Hochbauten – Erfahrungen und neue Erkenntnisse*. Ingenieurvermessung 84, Band 2, Dümmlers Verlag, Bonn

SCHRICK, K. W. (1977): *Anwendungen der Kalman-Filter-Technik.* R. Oldenbourg Verlag, München

WELSCH, D. / HEUNECKE, O. / KUHLMANN, H. (2000): *Auswertung geodätischer Überwachungsmessungen*. Handbuch Ingenieurgeodäsie, Wichmann Verlag, Heidelberg

ZURMÜHL, R. (1964): *Matrizen und ihre technischen Anwendungen*. Springer-Verlag, Berlin/ Göttingen/Heidelberg