



Presented at the FIG Working Week 2023,  
28 May - 1 June 2023 in Orlando, Florida, USA

# FIG WORKING WEEK 2023

28 May - 1 June 2023 Orlando Florida USA

Protecting  
Our World,  
Conquering  
New Frontiers

# Coordinates Trajectory Models

## Basics and Implementation

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## Outline:

- Project Motive
- Area of Interest
- Trajectory Models
- Project Implementation
  - GNSS data gathering and processing
  - Application of the SLTM
- Investigation of Outcome

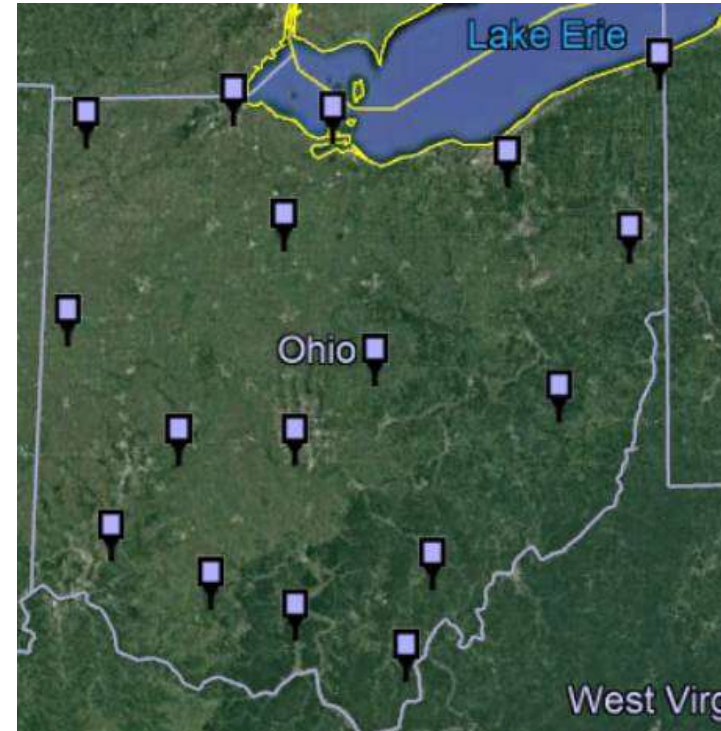
## Project motive

- Create a geodetic hands-on experience using techniques such as least squares adjustment, GNSS data, and timeseries analysis
- Implement trajectory modeling on more CORS stations.



## Area of Interest

- 16 CORS stations from the state of Ohio.



## Trajectory models

- Early scientists considered coordinates as fixed.
- Multiple trajectory models have evolved throughout history with development of theories and technologies.
- Each new model tries to fit and better represent the data.

- **Constant Velocity Model (CVM)**

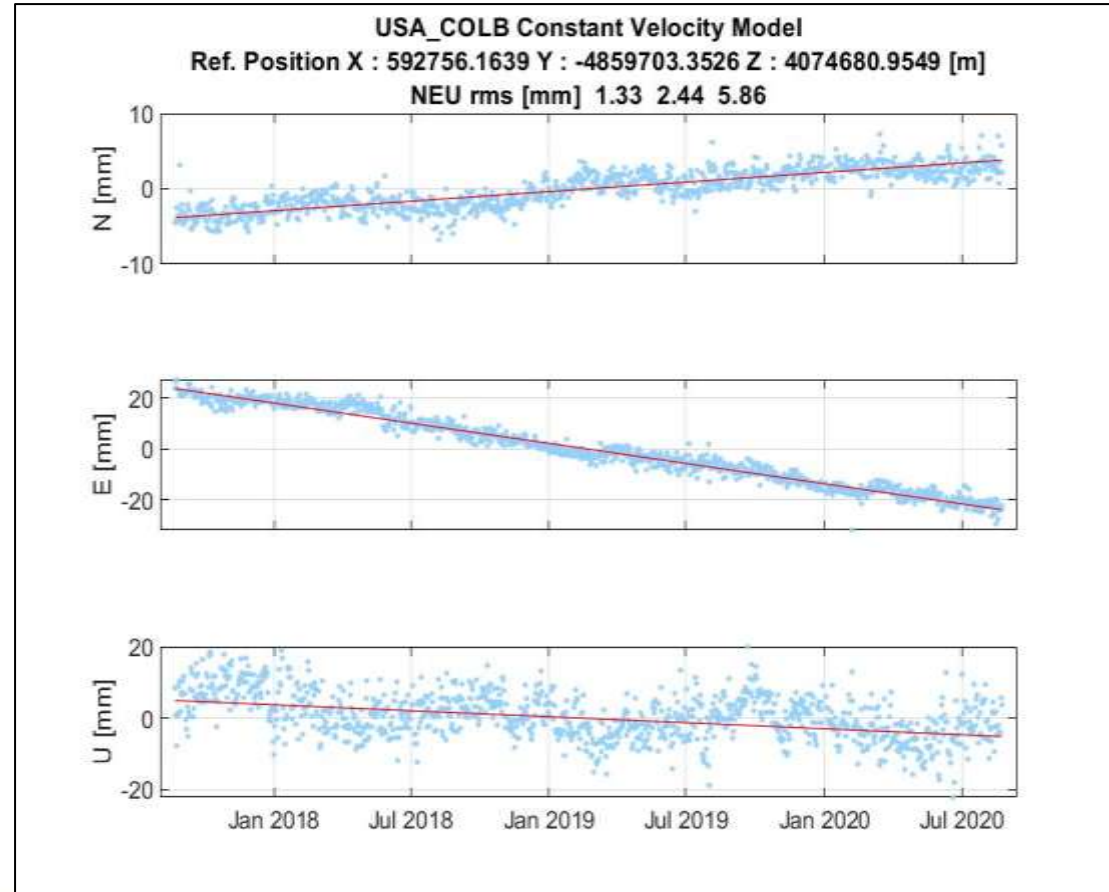
- $X(t) = X_R + V_x(t - t_R)$

- $Y(t) = Y_R + V_y(t - t_R)$

- $Z(t) = Z_R + V_z(t - t_R)$

- And in vector notation:

- $x(t) = x_R + v(t - t_R)$

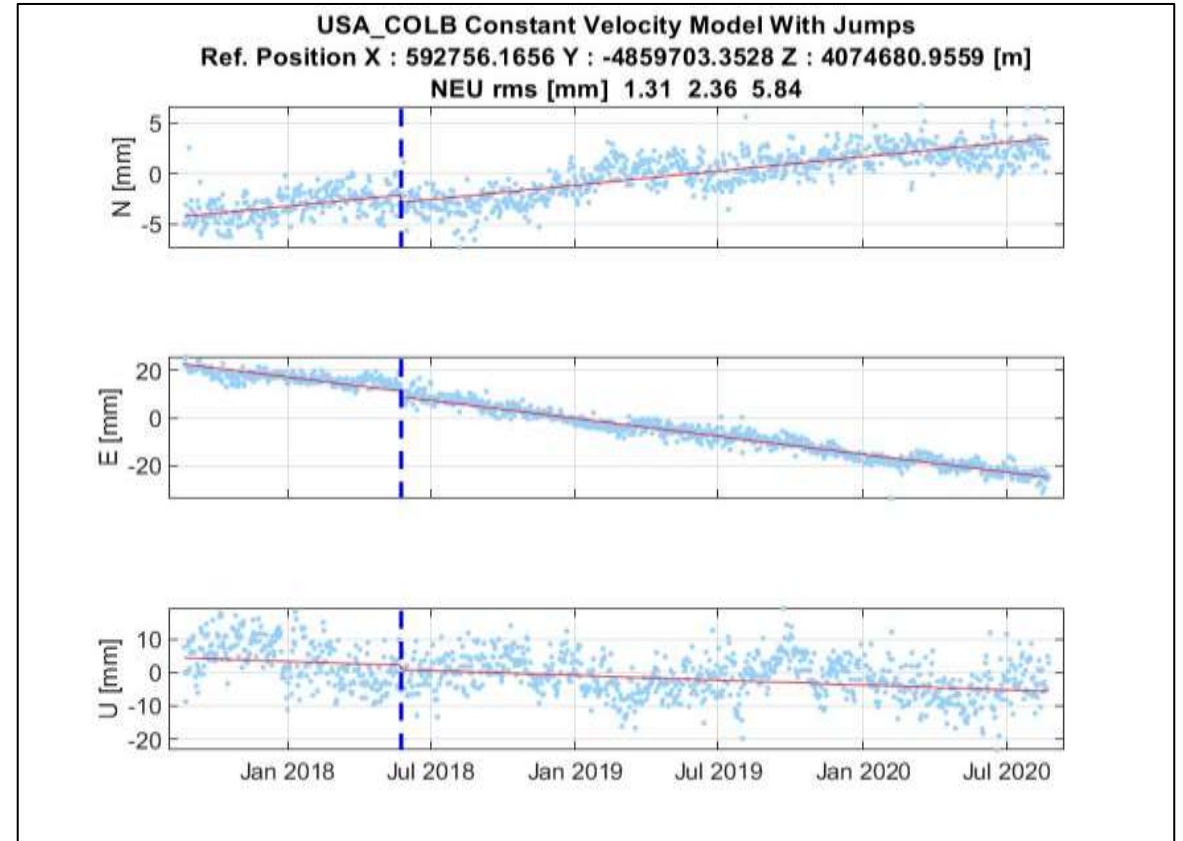




- Constant Velocity Model (CVM) with Jumps

- $x(t) = x_R + v(t - t_R)$

- $+ \sum_{j=1}^{n_j} b_j H(t - t_j)$

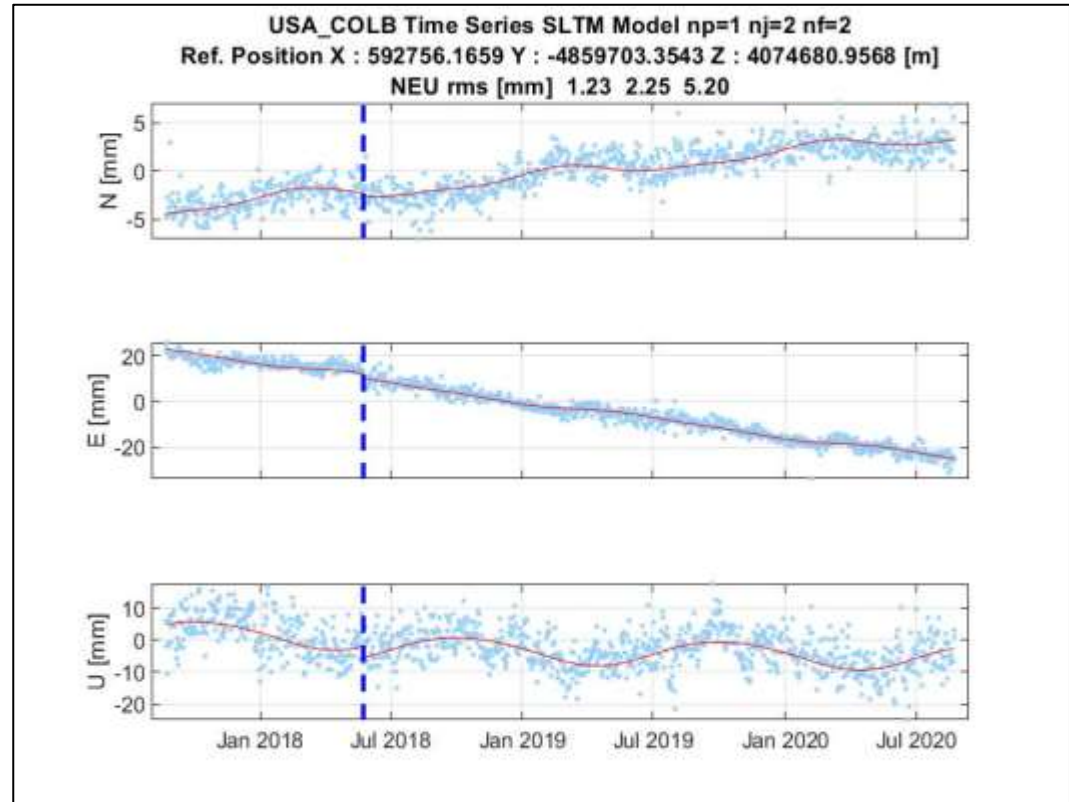


- Standard Linear Trajectory Model (SLTM)

- $x(t) = \sum_{i=1}^{np+1} p_i(t - t_R)^{i-1}$

- $+ \sum_{j=1}^{nj} b_j H(t - t_j)$

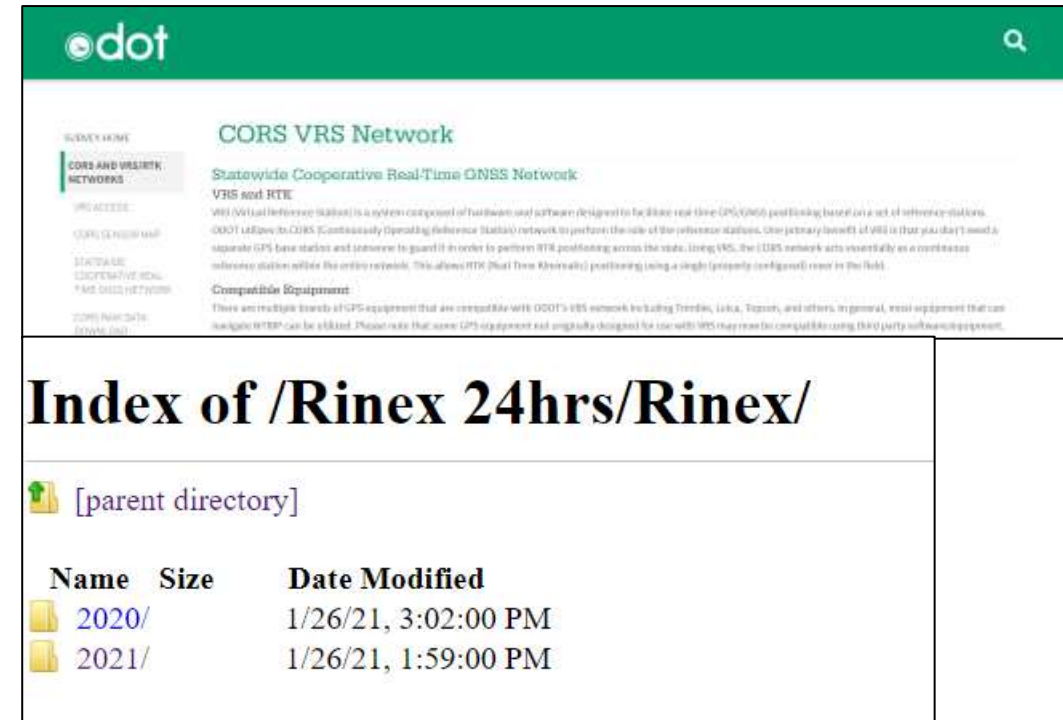
- $+ \sum_{k=1}^{nf} [s_k \sin(\omega_k t) + c_k \cos(\omega_k t)]$





- Extended Linear Trajectory Model (ELTM)
- $x(t) = \sum_{i=1}^{np+1} p_i(t - t_R)^{i-1} +$
- $\sum_{j=1}^{nj} b_j H(t - t_j) +$
- $\sum_{k=1}^{nf} [s_k \sin(\omega_k t) + c_k \cos(\omega_k t)] +$
- $\sum_{i=1}^{nT} a_i \log(1 + \Delta t_i / T_i)$
- \* We usually use two values for T where T1=1 and T2=0.0523

- **GNSS data processing**
  - Needed data is 3 years of data.
  - Out of the stations available in the OSU archive, 16 spread stations are selected
  - Obtained all RINEX files from for processing.



**dot**

**CORS VRS Network**  
Statewide Cooperative Real-Time GNSS Network

**VRS real-time**  
Virtual Reference Station (VRS) is a system composed of hardware and software designed to facilitate real-time (RT) GNSS positioning based on a set of reference stations. DOT utilizes its CORS Continuously Operating Reference Station network to perform the role of the reference stations. Our primary focus of VRS is that you don't need separate GPS base station and antenna to get RT positioning across the state. Using VRS, the CORS network acts essentially as a continuous reference station within the entire network. This allows RTK (Real Time Kinematic) positioning using a single (properly configured) rover in the field.

**Compatible Equipment**  
There are multiple brands of GPS equipment that are compatible with DOT's VRS network including Trimble, Leica, Topcon, and others. In general, most equipment that can accept NTRIP can be utilized. Please note that some GPS equipment not originally designed for use with VRS may need to be configured using third party software/equipment.

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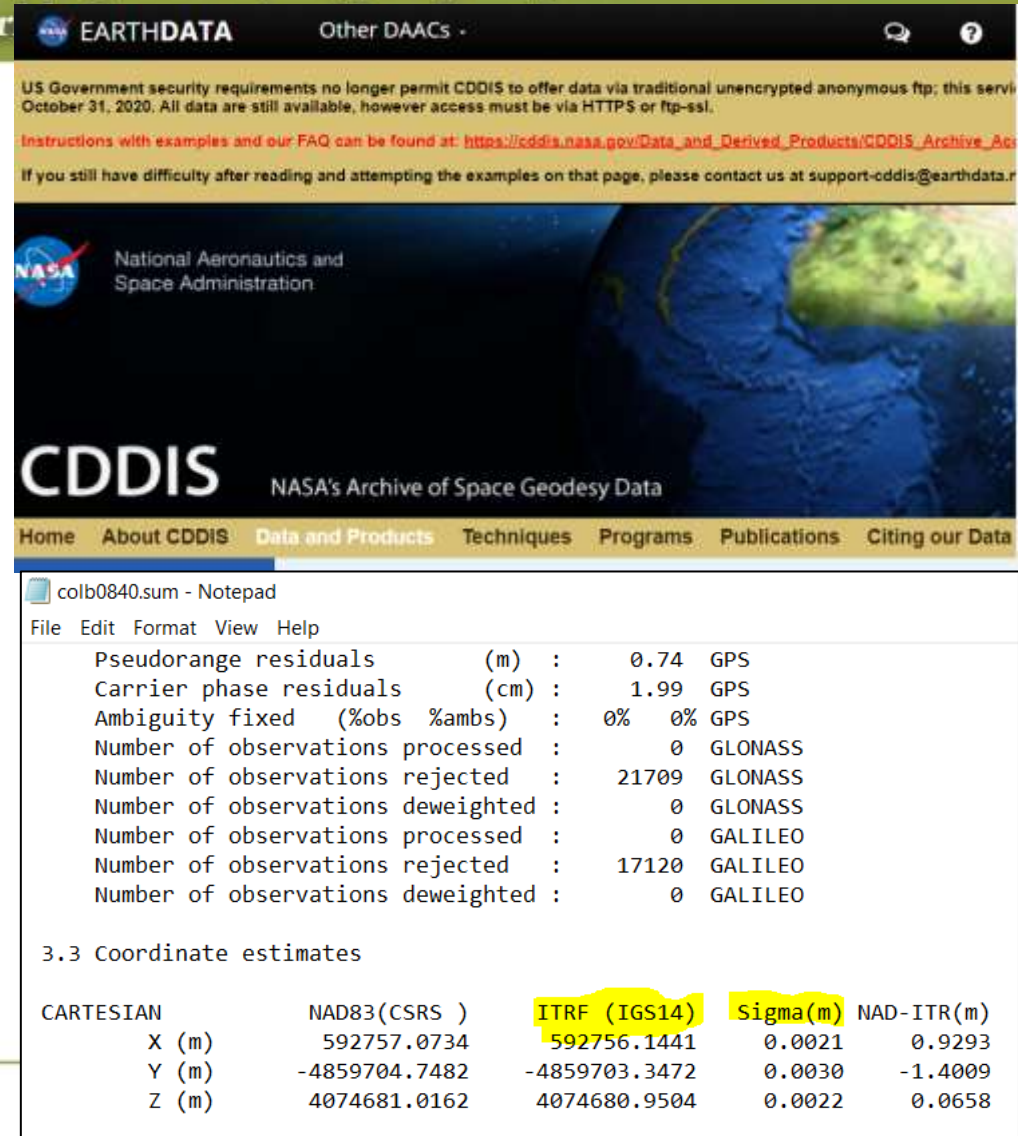
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Name	Size	Date Modified
2020/		1/26/21, 3:02:00 PM
2021/		1/26/21, 1:59:00 PM

- GNSS data processing

- Downloaded the GPSACE software along with required files such as precise ephemeris, clock correction, and earth rotation parameters files from NASA website



EARTHDATA Other DAACs

US Government security requirements no longer permit CDDIS to offer data via traditional unencrypted anonymous ftp; this service will be discontinued on October 31, 2020. All data are still available, however access must be via HTTPS or ftp-ssl.

Instructions with examples and our FAQ can be found at: [https://cddis.nasa.gov/Data\\_and\\_Derived\\_Products/CDDIS\\_Archive\\_Access](https://cddis.nasa.gov/Data_and_Derived_Products/CDDIS_Archive_Access)

If you still have difficulty after reading and attempting the examples on that page, please contact us at support-cddis@earthdata.nasa.gov

NASA National Aeronautics and Space Administration

## CDDIS

NASA's Archive of Space Geodesy Data

Home About CDDIS Data and Products Techniques Programs Publications Citing our Data

colb0840.sum - Notepad

File Edit Format View Help

```

Pseudorange residuals (m) : 0.74 GPS
Carrier phase residuals (cm) : 1.99 GPS
Ambiguity fixed (%obs %amb) : 0% 0% GPS
Number of observations processed : 0 GLONASS
Number of observations rejected : 21709 GLONASS
Number of observations deweighted : 0 GLONASS
Number of observations processed : 0 GALILEO
Number of observations rejected : 17120 GALILEO
Number of observations deweighted : 0 GALILEO
    
```

3.3 Coordinate estimates

CARTESIAN	NAD83(CSRS )	ITRF (IGS14)	Sigma(m)	NAD-ITR(m)
X (m)	592757.0734	592756.1441	0.0021	0.9293
Y (m)	-4859704.7482	-4859703.3472	0.0030	-1.4009
Z (m)	4074681.0162	4074680.9504	0.0022	0.0658



- Standard Linear Trajectory Model (SLTM)

- $x(t) = \sum_{i=1}^{np+1} p_i (t - t_R)^{i-1}$

- $+ \sum_{j=1}^{nj} b_j H(t - t_j)$

- $+ \sum_{k=1}^{nf} [s_k \sin(\omega_k t) + c_k \cos(\omega_k t)]$

- **Weighted Least Squares Adjustment**
- $\hat{x} = B^{-1}(A^T C_b^{-1} b)$
- $B = A^T C_b^{-1} A$
- $C_x = B^{-1}$
- A is the design or coefficient matrix
- $\hat{x}$  vector of unknown parameters

- **MATLAB Code Structure**

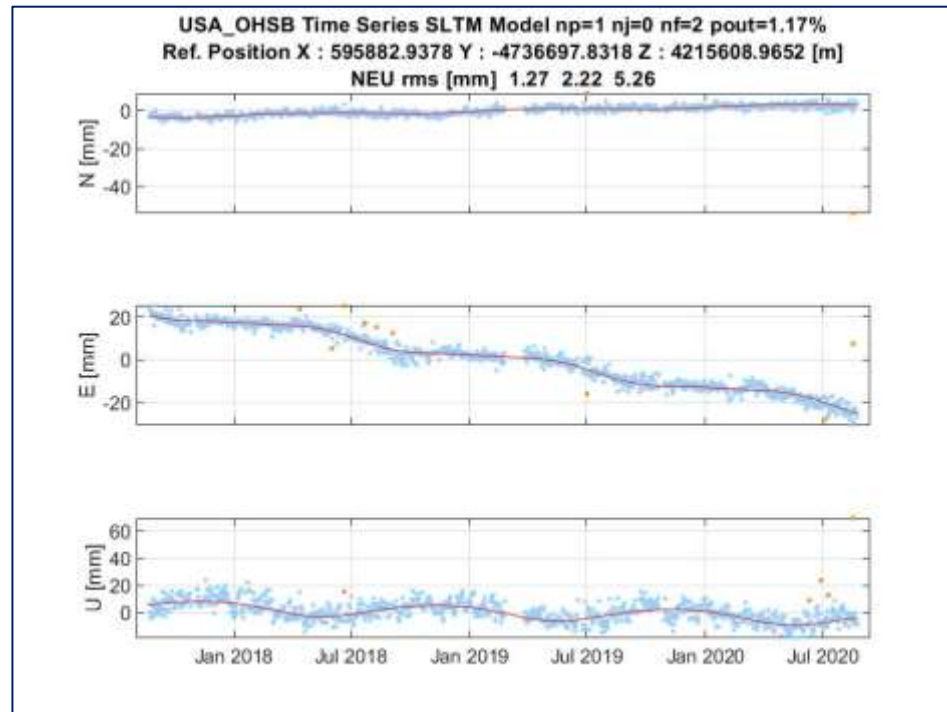
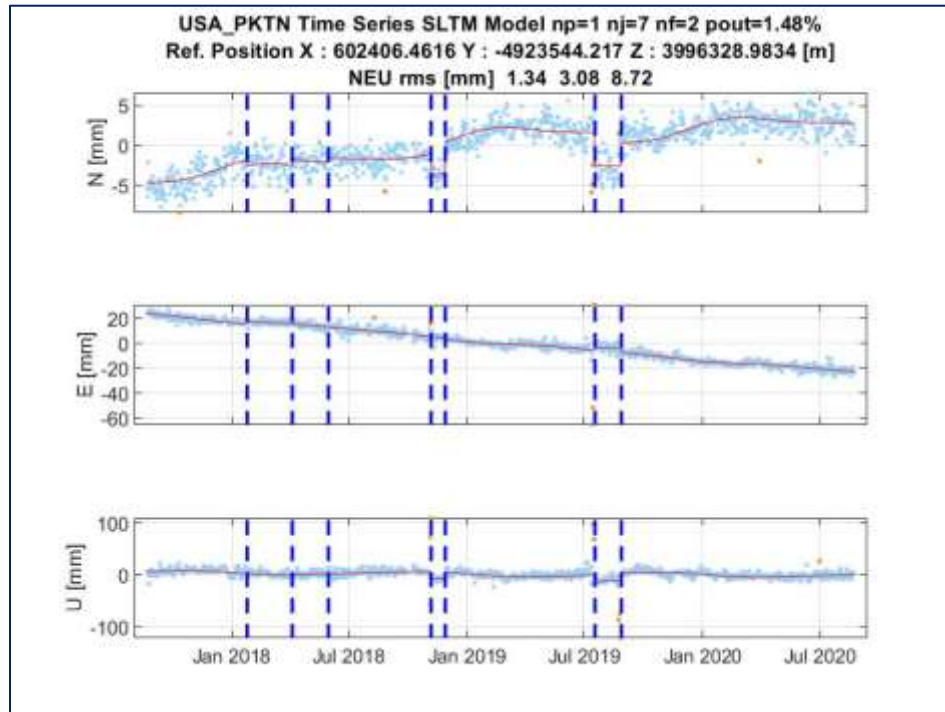
- Import coordinate time series with sigmas
- Import metadata such as jumps times.
- Generate the design matrix A
- Perform 1<sup>st</sup> lest squares to generate reference position
- Convert coordinates from xyz to enu.
- Perform 2<sup>nd</sup> least squares to estimate parameters of the model.



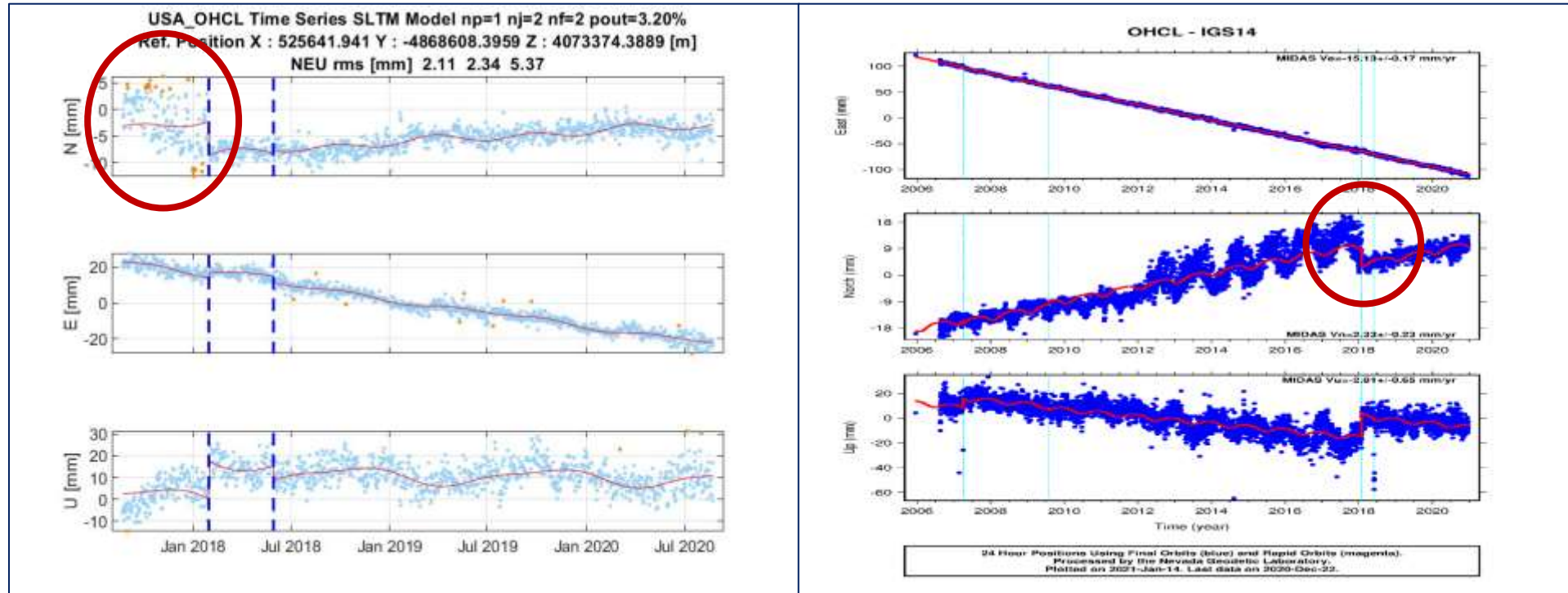
- **MATLAB Code Structure**

- Outlier detection and removal
- Perform 3rd least squares to estimate final parameters of the model.
- Use the design\_sltm to generate uniform time series.
- Plot the data.

- Investigation of outcome



- Investigation of outcome



\*Nevada Geodetic Laboratory Plot of OHCL



Thank you

**This is a Peer Reviewed Paper  
FIG Working Week 2023**

**Coordinates Trajectory Models: Basics and Implementation**

**Mohammed ALJAFAR, Saudi Arabia**