

The NGS Surface Gravity Prediction Tool

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SUMMARY

In support of a new vertical datum, the National Oceanic and Atmospheric Administration's (NOAA) National Geodetic Survey (NGS) is developing a number of completely new products and services. This vertical datum, the North American-Pacific Geopotential Datum of 2022 (NAPGD2022), will encompass a number of interrelated datasets including a geoid model, time-dependent geoid model, deflections of the vertical, surface gravity model, digital elevation model, etc. This paper will focus on the research and development of a new surface gravity prediction tool and methodology.

NGS has upwards of 10 million terrestrial gravity observations and 100 million GRAV-D airborne gravity measurements all observed at different epochs, by different observers, and with different techniques. This paper will investigate how these various datasets can be combined most effectively with their very different spatial wavelength characteristics with the goal of predicting a *consistent* full-field gravity value on the surface of the earth at an arbitrary location.

A number of separate gravity models with different methodology, input data, and assumptions will be evaluated including NGS's current surface gravity prediction tool, reference model enhanced with terrain based approaches, and interpolation schemes based on least squares collocation that utilize existing surface gravity data. These various methods will all be validated against external high accuracy gravity data acquired by the NGS Geoid Slope Validation Surveys (GSVS) in Texas and Iowa along with other high accuracy absolute gravity throughout the U.S. and its territories.

Results show that an interpolation scheme based on least squares collocation with a three-dimensional logarithmic covariance function performs the best in terms of residual RMS against the external validation datasets with 0.78 mGal on GSVS11, 1.56 mGal on GSVS14, 1.39 mGal with RELBASEA, and 0.65 mGal with GRAV-D control surveys. In comparison, the current surface gravity prediction tool performs at 1.44, 1.84, 2.27, and 1.88 mGal RMS, respectively, against the same external validation datasets.

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1. INTRODUCTION

For the past few decades, the NGS has utilized two different but very similar tools for the prediction of surface gravity at an arbitrary location. With the modernization of the NSRS, NGS has begun work to replace these tools to provide improved results and a more interactive experience for the user. This paper will focus on the first of these tasks and investigate how much improvement can be achieved over the existing Surface Gravity Prediction Tool with the advent of newer methods and availability of new data. To provide an external validation of the methods, a number of independent validation datasets have been developed that have high accuracy gravity observations along with positional information (latitude, longitude, and orthometric height). These datasets come from four different sources: GSVS11 (Smith, et al., 2013), GSVS14 (Wang, et al., 2017), RELBASEA (NGS's internal gravity database), and GRAV-D absolute gravity control surveys.

2. EXISTING NGS PRODUCTS FOR GRAVITY PREDICTION

NGS has two existing web-based tools that will predict a full field gravity value at a user provided location: 1) NAVD 88 gravity and 2) the Surface Gravity Prediction Tool. Two different tools that provide almost the same information are needed because they fulfill slightly different roles. The NAVD 88 gravity data and tool is designed to compute geodetic leveling corrections consistent with the current vertical datum, NAVD 88. It was determined using a gravity model at the time of the NAVD 88 adjustment. Any new or revised gravity values observed since that adjustment have not been incorporated into the NAVD 88 gravity and will have no impact on results. On the other hand, the Surface Gravity Prediction Tool (Fury, 1999) does incorporate new and revised gravity observations into its computations providing a gravity value that is a current reflection of the NGS's Integrated Database (IDB). Additionally, both of these tools are only valid for certain portions of the U.S. and will provide an error if you are outside of the computation region. Furthermore, the Surface Gravity Prediction tool completely fails to provide a value in situations where the uncertainty becomes too large. Both tools have been used for their respective purposes very successfully over the past ~20 years. However, they are both in need of a refresh in order to be coincident with NSRS modernization. As an example, the publically available interface for both tools is shown in Figure 1.

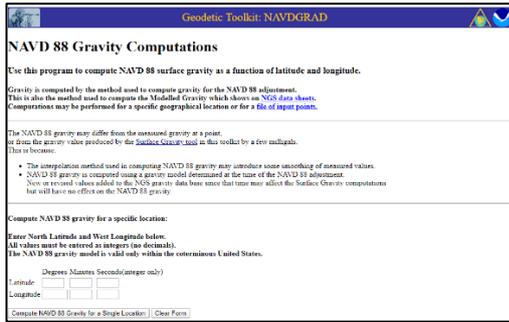


Figure 1: Existing NGS gravity interpolation web interfaces. a) (left) NAVD 88 Gravity (<https://www.ngs.noaa.gov/TOOLS/Navdgrav/navdgrav.html>) and b) (right) NGS Surface Gravity Prediction Tool Interface (https://www.ngs.noaa.gov/cgi-bin/grav_pdx.prl).

For added context, Figure 2 provides the output from the two existing tools evaluated at NGS's Table Mountain Geophysical Observatory (TMGO) outside Boulder, Colorado. Absolute gravity observations are taken here on a roughly weekly basis so the actual gravity value on any given pier at this facility is known to a high degree of accuracy. The best estimate of the average gravity value at this facility is 979622.744 ± 0.002 mGals.

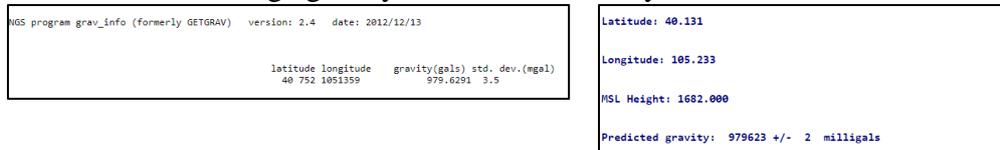


Figure 2: Output from two existing NGS gravity prediction tools. a) (left) NAVD 88 Gravity. b) (right) Surface Gravity Prediction.

3. METHODS

Over the last 50 years in geodesy, numerous ways to predict the gravity value at an arbitrary location have been studied (Hardy and Gopfert, 1975; Priovolos, 1988; Filmer, et al. 2013). All of these methods rely on the following data to some degree or another: 1) observed surface gravity data which includes latitude, longitude, elevation, and full-field gravity; and 2) a Digital Elevation Model (DEM). In different scenarios, the observed surface gravity data may be incorporated into a reference model, gridded, or used as discrete points.

A brief mention should be made about different gravity related anomaly fields. Our goal is to predict the full field gravity value (e.g. 978000 mGal). Any relevant anomaly fields may be used in the prediction, but must be converted back to the full field gravity at the end. Usually, this can be done with knowledge about the elevation of the prediction point. In the event, the prediction point elevation is unknown, a DEM can be used but injects additional error into the gravity prediction.

3.1 Reference Model Enhanced with Terrain

This method consists of a geopotential reference model that accurately predicts the long wavelength features of the gravity field that is augmented with gravity derived from topographic information at the high frequencies. This method is advantageous due to the plethora of geopotential reference models available and their use as the basis for many global

and regional geoid models. The predicted surface gravity value (\hat{g}) can be determined from (1) where the term in parenthesis is the predicted free-air gravity and the other terms simply ‘restore’ the full field gravity from the free-air gravity.

$$\hat{g} = (g_{REF}^{2-2160} + g_{topography}^{2161+}) + \frac{d\gamma}{dh}H + \frac{1}{2} \frac{d^2\gamma}{dh^2}H^2 + \gamma \quad (1)$$

where:

g_{REF}^{2-2160} = synthesized free-air gravity from a reference model from degree 2 to 2160

$g_{topography}^{2161+}$ = high-frequency gravity contribution from topography from degree 2161 to resolution of terrain model

$$\gamma = \frac{a\gamma_a \cos^2 \varphi + b\gamma_b \sin^2 \varphi}{\sqrt{a^2 \cos^2 \varphi + b^2 \sin^2 \varphi}}; \frac{d\gamma}{dh} = \frac{-2\gamma}{a} (1 + f + m - 2f \sin^2 \varphi); \frac{d^2\gamma}{dh^2} = \frac{6\gamma}{a}$$

The spherical harmonic coefficients from a geopotential reference model can be used to synthesize the reference model portion of (1) from degree 2 to the maximum degree of the model (e.g. 2160). We make use of two different NGS reference models: REF16A and REF19B (Li, et al. 2019). These two models are identical except for the inclusion of GRAV-D airborne data in REF19B. The topographic portion can be used directly from existing models like ERTM2160 (Hirt, et al., 2014) and SRTM2gravity (Hirt, et al., 2019) or computed from a terrain model. In this investigation, we use both ERTM2160 to ~250 m resolution and SRTM2gravity to ~90 m resolution.

3.2 Refined Bouguer Anomaly

In this method, the refined Bouguer anomaly is used as the basis for prediction. We use the term ‘refined’ to signify that a terrain correction has been applied to the (simple) Bouguer anomaly. On a point-by-point basis, the refined Bouguer anomaly is computed from (2).

$$\Delta g = g - \frac{d\gamma}{dh}H - \frac{1}{2} \frac{d^2\gamma}{dh^2}H^2 - \gamma + \delta g_{ATM} - 0.11195H + A \quad (2)$$

where:

A = standard terrain correction with density of 2.67 g/cm³

$\gamma_a, \gamma_b, a, b, f, m$ are all parameters from the GRS80 reference ellipsoid

$$\delta g_{ATM} = 0.87 * e^{-0.116*(H/1000)^{1.047}}$$

In order to compute the A term in (2), we make use of the TC program (Forsberg and Tscherning, 2008) using a detailed 3” DEM to 50 km and a 30” DEM to an outer zone distance of 300 km both based on the USGS NED DEM (Gesch, et al., 2009).

There are two ways in which the point-by-point anomaly data computed in (2) can be used in the prediction: 1) predict on a regular grid (e.g. 1’) and then interpolate from this grid at the user specified location; and 2) predict exactly at the user specified location. The former has a slightly lower computational ‘overhead’ from a user’s perspective as the regular grid is computed a priori and the prediction is nothing more than an interpolation. This also makes it convenient if the desire is to keep the identical gravity field constant for many years (i.e. the NGS NAVD 88 gravity). However, based on preliminary tests, omission and degradation in

the overall results are likely to make this scheme less desirable compared to the latter. Consequently, this scheme will not be presented in intricate detail.

In the exact point prediction scheme, least squares collocation (Moritz, 1980) is employed with surrounding surface gravity data. For computational efficiency, only data within 50 km of the evaluation point are used to determine the best-fit plane, which is subtracted from the data. From this plane-removed anomaly data, the nearest 15 points in each geographic quadrant are ultimately used in the prediction (i.e. 60 points total are used in the prediction).

The covariance function used is shown in (3) and is based on a three-dimensional logarithmic function (Forsberg, 1987) that uses location specific parameters (D and T) depending on the existing surface gravity data (latitude, longitude, and height).

$$C(\Delta g^{H_1}, \Delta g^{H_2}) = -f \sum_{i=0}^3 \alpha_i \log(z + r) \quad (3)$$

where:

$f = C_0 / \log\left(\frac{D_1^3 D_3}{D_0 D_2^3}\right)$, with $D_i = D + iT$ and C_0 as the variance.

$\alpha_0 = 1; \alpha_1 = -3; \alpha_2 = 3; \alpha_3 = -1;$

$z = H_1 + H_2 + D_i; r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + z^2}$

This method has three primary advantageous compared to other methods/covariance functions. First, the three-dimensional nature of the gravity field is kept intact with a three-dimensional covariance function. Second, location specific parameters allow for fine-tuning of the prediction in different regions (i.e. not a one-size fits all solution). Third, while beyond the scope of this paper, the three-dimensional logarithmic covariance function can easily be adapted to other geopotential related quantities like deflections of the vertical, gravity gradients, etc.

The selection of parameters (D and T) is done based on a leave-one-out prediction on the existing data points where the combination of D and T that fits the existing surface data with the lowest RMS is selected for that evaluation point. In order to avoid combinations of D and T that have no real-world basis, the parameters are restricted to be $D = [2 \text{ km}, 4 \text{ km}, 6 \text{ km}, 8 \text{ km}, 10 \text{ km}, 12.5 \text{ km}, 15 \text{ km}, 20 \text{ km}]$ and $T = [30 \text{ km}, 60 \text{ km}, 90 \text{ km}, 120 \text{ km}]$.

Once the prediction has occurred at the user specified location, the predicted full field gravity (\hat{g}) is restored from the *predicted* refined Bouguer anomaly ($\widehat{\Delta g}$) at the user location according to (4), where the additional ‘restore’ terms are all determined from the user specified location.

$$\hat{g} = \widehat{\Delta g} + \frac{dy}{dh}H + \frac{1}{2} \frac{d^2\gamma}{dh^2}H^2 + \gamma - \delta g_{ATM} + 0.11195H - A \quad (4)$$

3.3 Refined Bouguer Anomaly Enhanced with GRAV-D airborne data

This method is identical to the previous method but has one significant adaptation. The existing surface gravity data is enhanced with relevant gravity information from airborne

GRAV-D data. To do this in the appropriate spatial wavelengths, the internally developed geopotential reference models, REF16A and REF19B, are used from degree 2 to 2190. The only difference between these two reference models is the incorporation of GRAV-D data in REF19B, spectrally. Therefore, in the free-air anomaly field, we remove the REF16A contribution and add back the REF19B contribution resulting in GRAV-D enhanced free-air anomaly as shown in (5) to (7). This is then used to compute the refined Bouguer anomaly as in (2) and the prediction follows exactly as the previous method. In preliminary tests, this adaptation has been shown to provide a slightly better fit than other methods.

$$\Delta g_{free-air}^{original} = g - \frac{d\gamma}{dh}H - \frac{1}{2} \frac{d^2\gamma}{dh^2}H^2 - \gamma + \delta g_{ATM} \quad (5)$$

$$\Delta g_{free-air}^{enhanced} = \Delta g_{free-air}^{original} - \Delta g_{free-air}^{REF16A} + \Delta g_{free-air}^{REF19B} \quad (6)$$

$$\Delta g = \Delta g_{free-air}^{enhanced} - 0.11195H + A \quad (7)$$

4. RESULTS

To assess the quality of the various prediction methods, we use four different high-accuracy gravity datasets in a validation scheme. These dataset can be thought of as ‘ground-truth’ as their gravity value is known to a high degree of accuracy. In theory, all of these datasets could be combined into a single validation but since they have different underlying observation techniques, uncertainties, and other caveats, they are kept separate from one another. Due to the unpublished nature of GSVS17 in Colorado, we don’t show those results here. Most of the validation gravity observations are determined by an FG-5 or A10 gravimeter. In GSVS11 and GSVS14, high accuracy relative gravity was observed but is based on very dense absolute gravity observations at every 10th station. The overall prediction results are shown in Table 1.

Generally, the primary interest will be in the residual RMS; however, in certain comparisons, the occurrence of a large mean (bias) significantly increases the RMS. Controlling the bias is rather problematic in some of these methods and subject to further investigation. Additionally, a bias over a fairly local area such as the GSVS14 profile is not necessarily alarming as the existing surface gravity data may be at fault. However, a bias in a CONUS-wide comparison should be carefully investigated. All of the following figures and statistics are based on the residual gravity of the model prediction (true gravity – predicted gravity) at the individual point locations (latitude, longitude, H).

Table 1: Overall prediction residual statistics for methods evaluated (best RMS is **bolded**)

	Num. Pts.	Method	Min	Max	Mean	Std. Dev.	RMS
GSVS11	218	Existing Surface Gravity Prediction	-3.32	0.56	-1.26	0.68	1.44
		REF16A+ERTM2160	-3.36	2.27	-0.42	1.04	1.12
		REF19B+ERTM2160	-1.65	3.74	0.37	0.88	0.95
		REF16A+SRTM2gravity	-3.47	1.95	-0.45	1.01	1.11
		REF19B+SRTM2gravity	-1.72	3.23	0.34	0.83	0.89
		Refined Bouguer: Exact	-2.59	1.61	-0.29	0.73	0.78
		GRAV-D Enhanced: Exact	-1.57	3.18	0.49	0.92	1.04

GSVS14	204	Existing Surface Gravity Prediction	-3.90	8.74	0.09	1.83	1.84
		REF16A+ERTM2160	-4.54	4.03	-0.39	1.50	1.55
		REF19B+ERTM2160	-4.16	3.99	-0.37	1.46	1.50
		REF16A+SRTM2gravity	-4.15	4.48	-0.44	1.55	1.61
		REF19B+SRTM2gravity	-3.85	4.24	-0.42	1.50	1.56
		Refined Bouguer: Exact	-5.58	4.31	0.40	1.51	1.56
		GRAV-D Enhanced: Exact	-5.82	4.46	0.40	1.55	1.60
RELBASEA	273	Existing Surface Gravity Prediction	-16.04	8.05	-0.76	2.13	2.27
		REF16A+ERTM2160	-10.29	8.66	-0.24	3.05	3.06
		REF19B+ERTM2160	-10.33	8.54	-0.30	3.03	3.05
		REF16A+SRTM2gravity	-11.30	8.24	-0.30	3.19	3.21
		REF19B+SRTM2gravity	-11.34	8.27	-0.36	3.17	3.19
		Refined Bouguer: Exact	-6.62	7.29	0.10	1.39	1.39
		GRAV-D Enhanced: Exact	-6.66	7.29	0.03	1.52	1.52
GRAV-D	33	Existing Surface Gravity Prediction	-4.39	2.22	-1.43	1.22	1.88
		REF16A+ERTM2160	-5.88	1.24	-2.05	1.79	2.72
		REF19B+ERTM2160	-5.76	1.20	-2.17	1.74	2.78
		REF16A+SRTM2gravity	-9.13	0.80	-2.45	2.23	3.31
		REF19B+SRTM2gravity	-9.13	0.82	-2.57	2.20	3.38
		Refined Bouguer: Exact	-1.46	1.32	-0.30	0.57	0.65
		GRAV-D Enhanced: Exact	-1.96	1.32	-0.44	0.68	0.80

4.1 GSVS11

This validation line in southern Texas is roughly 325 km long and consists of 218 stations with accurate GNSS, leveling, and gravity on each station. It has a very smooth gravity field to predict with a peak-to-peak range of only ~50 mGals in the Bouguer anomaly field. This smoothness causes the prediction results for all methods to be very accurate (RMS of ~1 mGal or less). The existing surface gravity tool performs only slightly worse but is still at 1.44 mGal. In addition to comparing and relying on a single RMS value, Figure 3 illustrates the percentage of stations that are within a particular threshold of the true gravity value (e.g. percentage within 0.5 mGal) to provide a little more nuance of the actual prediction distribution. The two best prediction models are the REF19B + SRTM2gravity method and the refined Bouguer anomaly method.

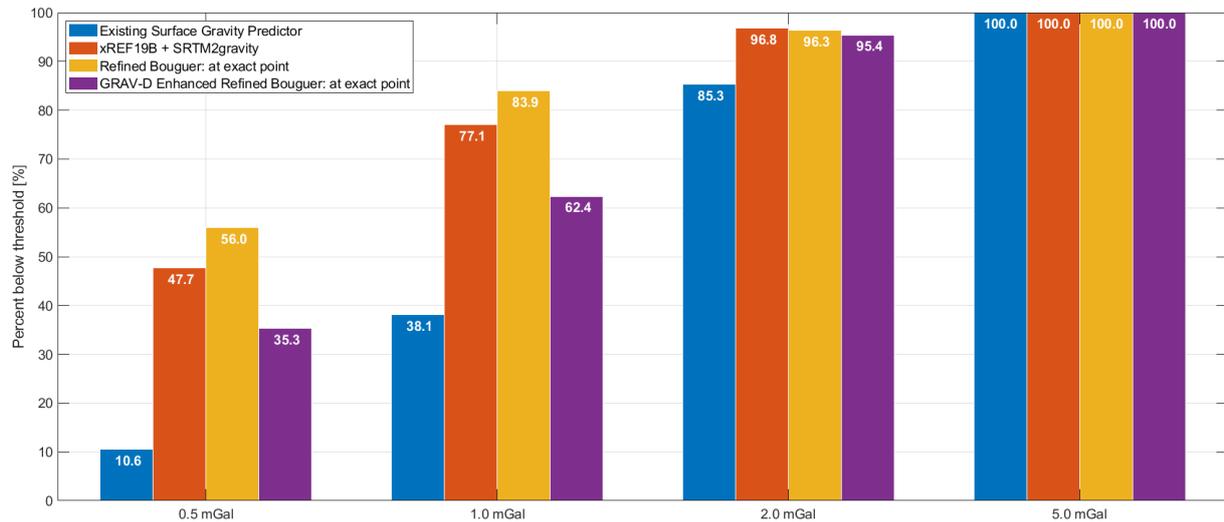


Figure 3: Percentage of GSVS11 stations within certain threshold for various prediction methods

The variants of the reference model method results are illustrated in Figure 4 with two major conclusions. First, the REFA model results are all affected by a tilt, which is not present in the REFB results. This illustrates the improvement (~0.2 mGal RMS) provided by the GRAV-D data to this method over Texas. Secondly, the updated SRTM2gravity high-frequency gravity information is only slightly better (0.05 mGal RMS) than the ERTM2160 data.

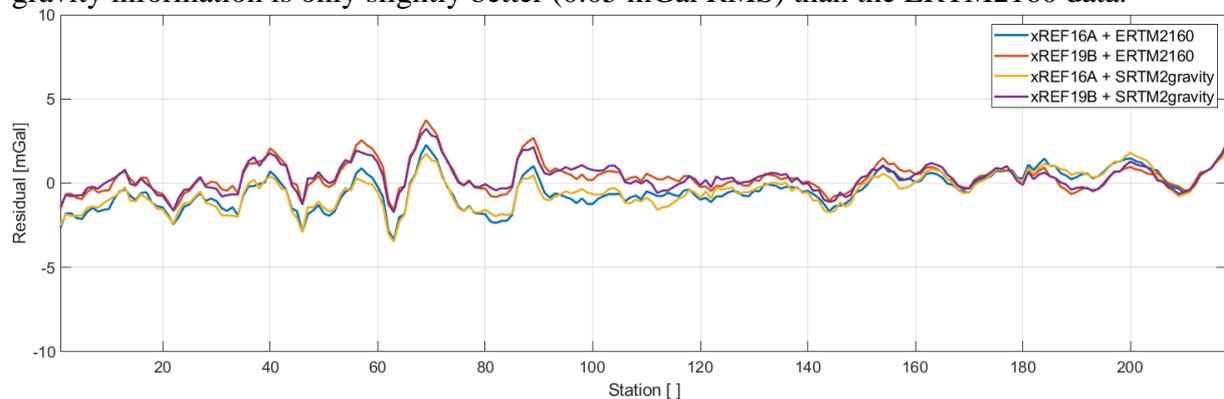


Figure 4: GSVS11 station prediction residuals for variants of the reference model enhanced with terrain

The results for the existing surface gravity prediction tool, the refined Bouguer method, and the GRAV-D enhanced method are shown in Figure 5. These results are overall quite good and fairly consistent. The best results are those obtained with the refined Bouguer method with an RMS of 0.78 mGal.

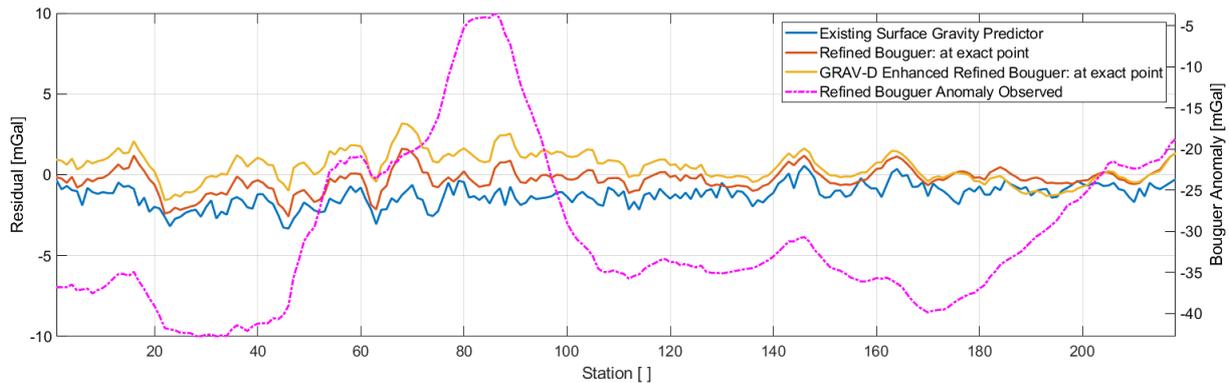


Figure 5: GSVS11 station prediction residuals for prediction methods that rely on the existing surface gravity data

4.2 GSVS14

This line in central Iowa is quite similar to the previously discussed GSVS11 profile in construction; however, the gravity field is much more undulating (peak-to-peak of ~150 mGal) due to the existence of the mid-continental rift. The overall prediction results reflect this more inconsistent gravity field with the best prediction methods performing at the 1.5 mGal RMS level. Surprisingly, all the prediction methods have RMS values between 1.5 to 1.6 mGal. The existing surface gravity prediction tool is slightly worse than this at 1.84 mGal. The results are illustrated in Figures 6 to 8.

While the REF19B + ERTM2160 method has the lowest overall RMS at 1.50 mGal, the refined Bouguer anomaly prediction method has a slightly better prediction distribution (5% more stations within 0.5 mGal and 7% more stations within 1 mGal) as illustrated in Figure 6. Additionally, there are some noticeable locations that have difficulty in the majority of prediction methods. From Figure 7 and 8, it is evident that there is a +/- 4 mGal prediction error around Stations 30 and 170. This is due to the steepness in the Bouguer anomaly in this area (see Figure 8) and the lack of existing surface gravity data to accurately predict the field.

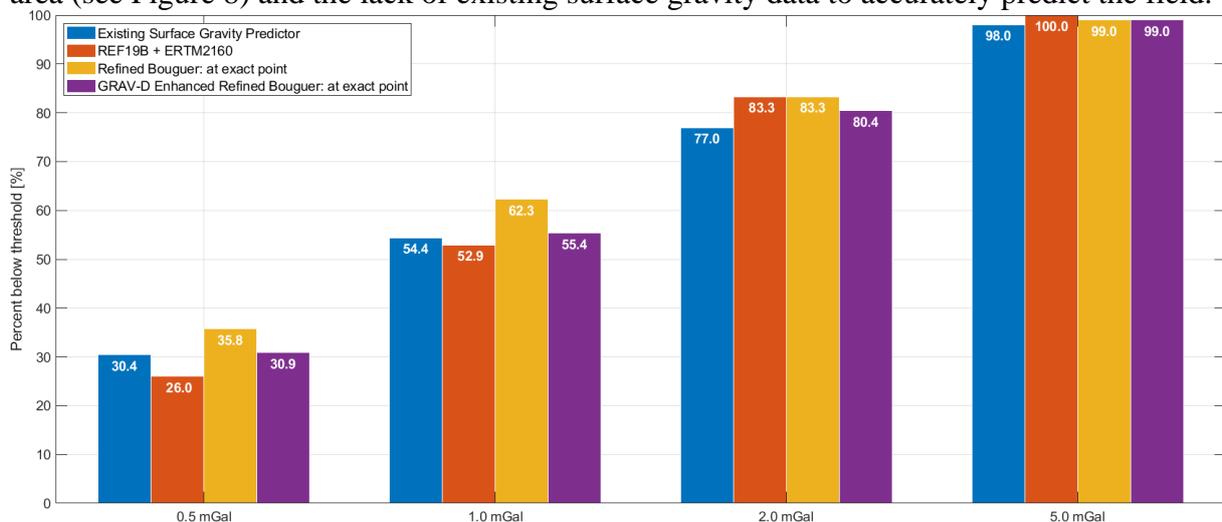


Figure 6: Percentage of GSVS14 stations within certain thresholds for various prediction methods

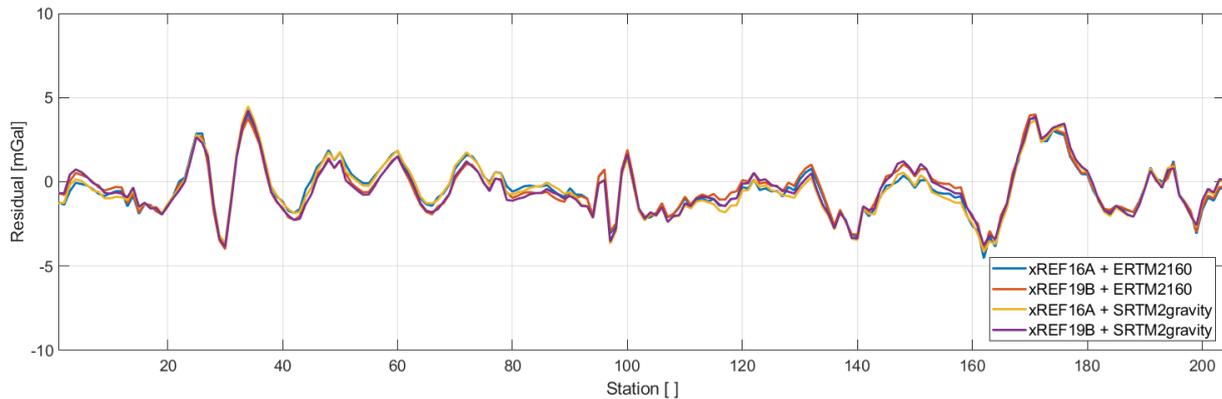


Figure 7: GSVS14 station prediction residuals for variants of the reference model enhanced with terrain

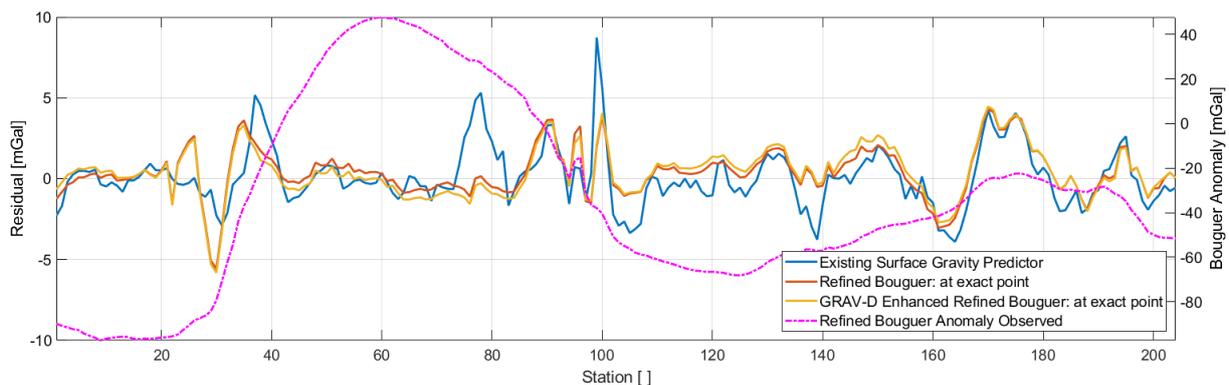


Figure 8: GSVS14 station prediction residuals for prediction methods that rely on the existing surface gravity data

4.3 Absolute Gravity Observations throughout CONUS

NGS has a small number of absolute gravity observations at discrete points that have been collected over many decades and maintained in the RELBASEA database. These stations have a wide range of accuracy in the orthometric height, ranging from scaled elevations from a topographic map to static GPS observations, which needs to be considered when evaluating the results of this validation dataset. As this data has been in existence for many decades, many of the observations are also in the NGS point-by-point data used in the prediction. To ensure that independence is maintained between the two datasets, the absolute gravity data (control) was evaluated with respect to the point-by-point data and any duplications were removed. The remaining dataset is approximately 273 stations throughout CONUS as shown in Figure 9a. NGS also has a slightly newer set of absolute gravity observations that have been collected to serve as control for the GRAV-D airborne project. These GRAV-D control observations all have GPS derived ellipsoid heights associated with them along with additional metadata. These are also completely independent from any observations in the point-by-point data used for the prediction. There are approximately 33 stations throughout CONUS in this dataset as shown in Figure 9b.

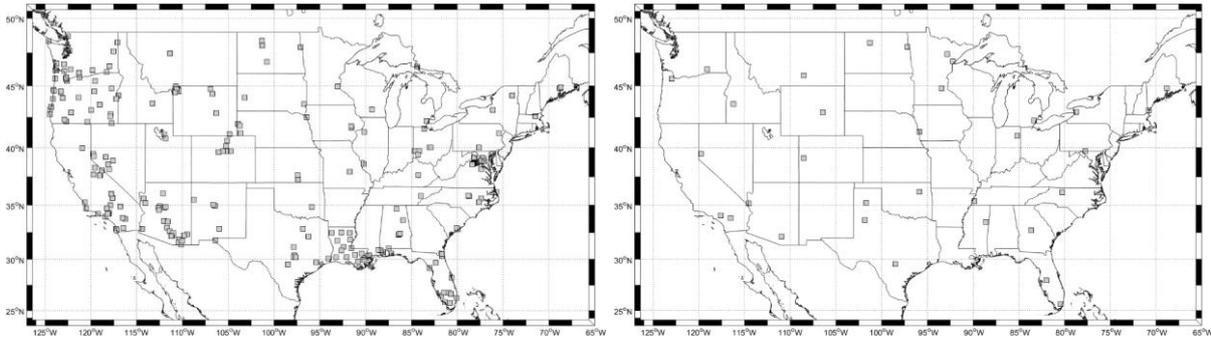


Figure 9: a) (left) RELBASEA absolute gravity validation station distribution in CONUS ($n = 273$). b) (right) GRAV-D absolute gravity validation station distribution in CONUS ($n = 33$).

4.3.1 RELBASEA Dataset Results

This dataset provides the most representative coverage of any of the validation datasets with a fairly uniform sampling across all of CONUS. There is an adequate number of stations (273) in a wide variety of environments (rugged and smooth terrain; varying gravity field situations). Results are illustrated in Figures 10 to 12. The refined Bouguer anomaly prediction method performs much better than the other methods with an overall RMS of 1.39 mGal. This is almost 0.9 mGal better than the existing surface gravity prediction tool. Like the GSVS14 results, this outperformance is present throughout the distribution compared to the next best model (10% more stations within 0.5 mGal; 4% more within 1 mGal; and 2% within 2 mGal). Surprisingly, all of the reference model prediction variants perform quite poorly with results at the 3.0+ mGal level. The cause of this poor performance may be due to inexact heights of the validation dataset, omission of the reference model, and/or lack of consistency between the high-frequency gravity field and the terrain; however, further investigation is needed.

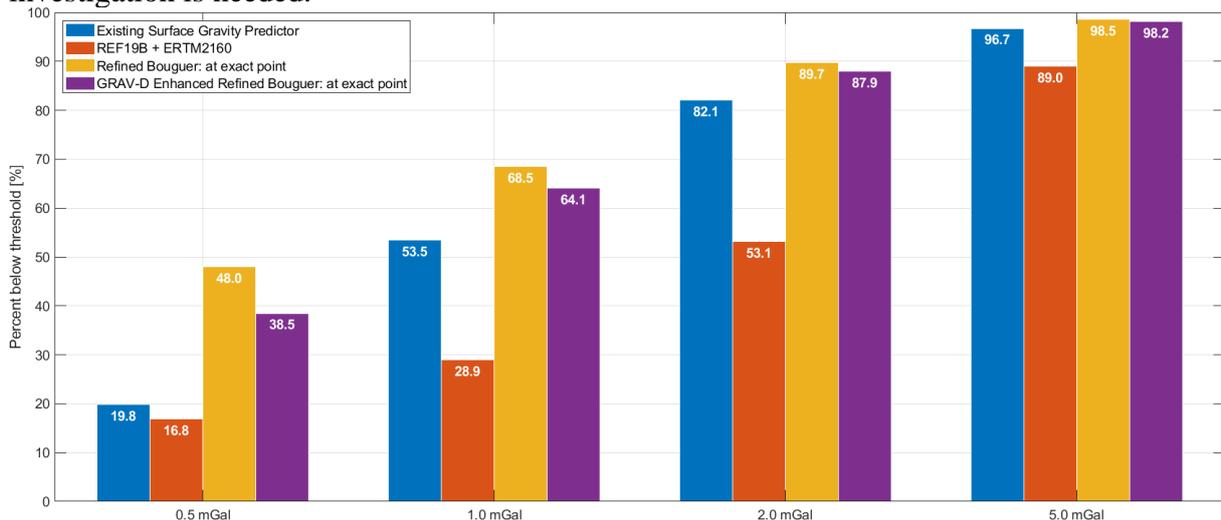


Figure 10: Percentage of RELBASEA stations within certain thresholds for various prediction methods

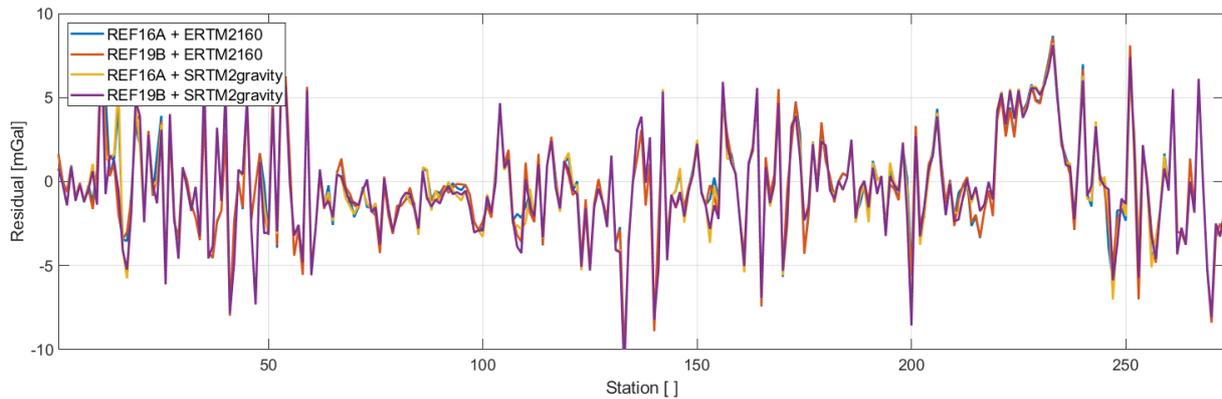


Figure 11: RELBASEA station prediction residuals for variants of the reference model enhanced with terrain

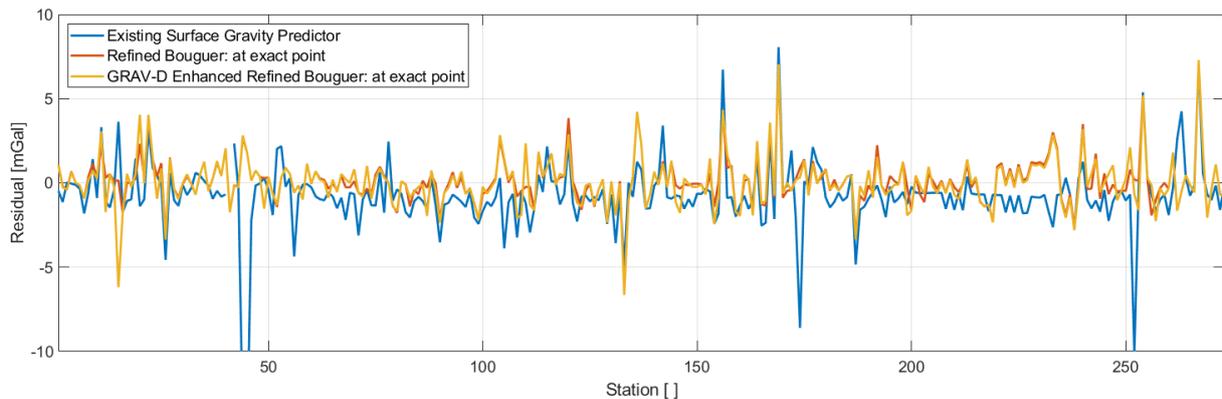


Figure 12: RELBASEA station prediction residuals for prediction methods that rely on the existing surface gravity data

4.3.2 GRAV-D Absolute Gravity Control Stations Results

This validation dataset consists of 33 stations with a reasonably uniform distribution across CONUS. Each station has a high accuracy height from GPS and gravity from an absolute gravimeter, which should be almost as accurate as the GSVS profiles. The only caveats with this dataset from a validation perspective is that there are only 33 stations, and they are all located at large airports with a fairly uniform terrain and gravity field in the surrounding area. Consequently, the prediction results are quite good with the refined Bouguer anomaly method having the best RMS at 0.65 mGal. This is an improvement of 1.2 mGal over the existing surface gravity prediction tool. Like the previous RELBASEA results, all of the reference model methods have quite a significant drop-off in performance (~2.7+ mGal RMS). Results are illustrated in Figures 13 to 15.

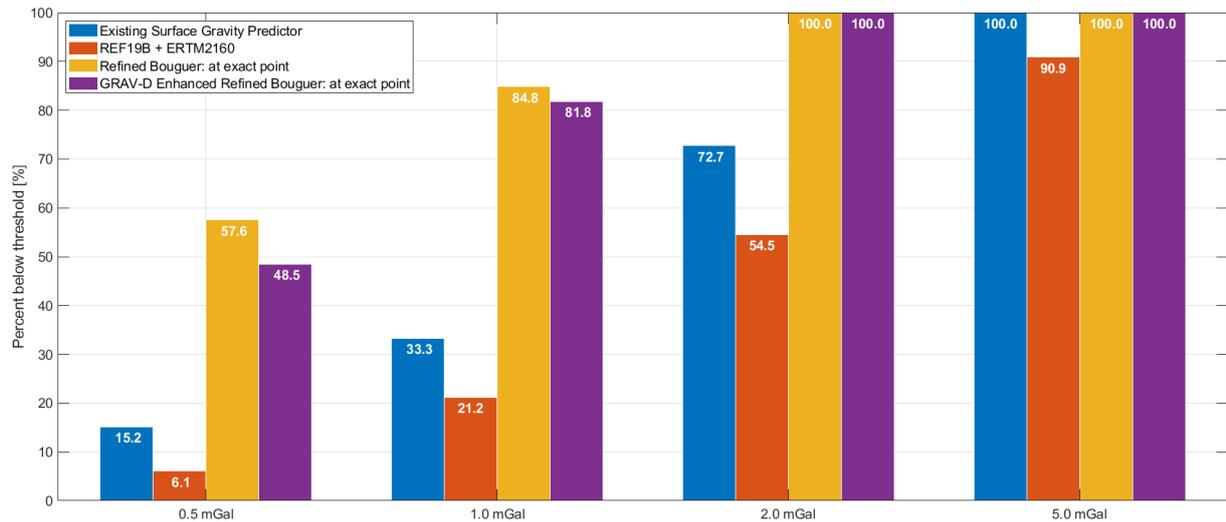


Figure 13: Percentage of GRAV-D control stations within certain thresholds for various prediction methods

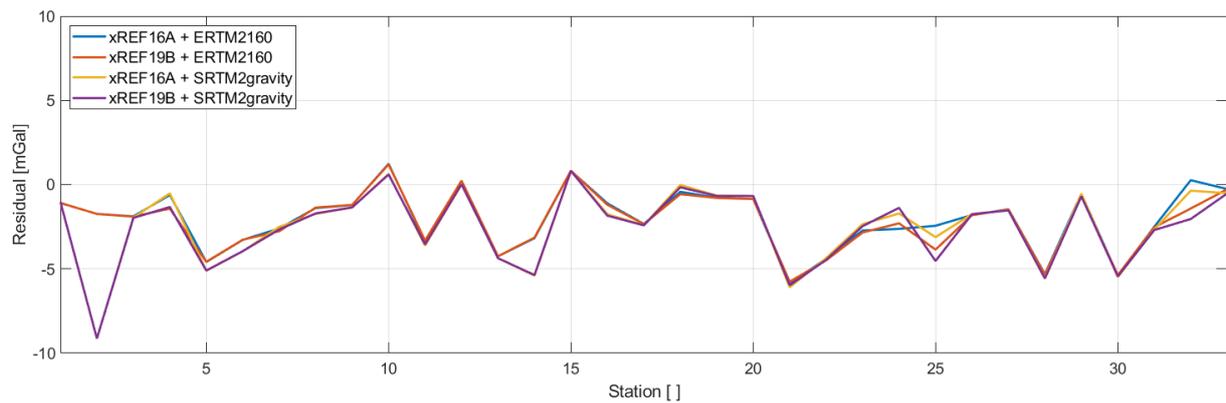


Figure 14: GRAV-D control station prediction residuals for variants of the reference model enhanced with terrain

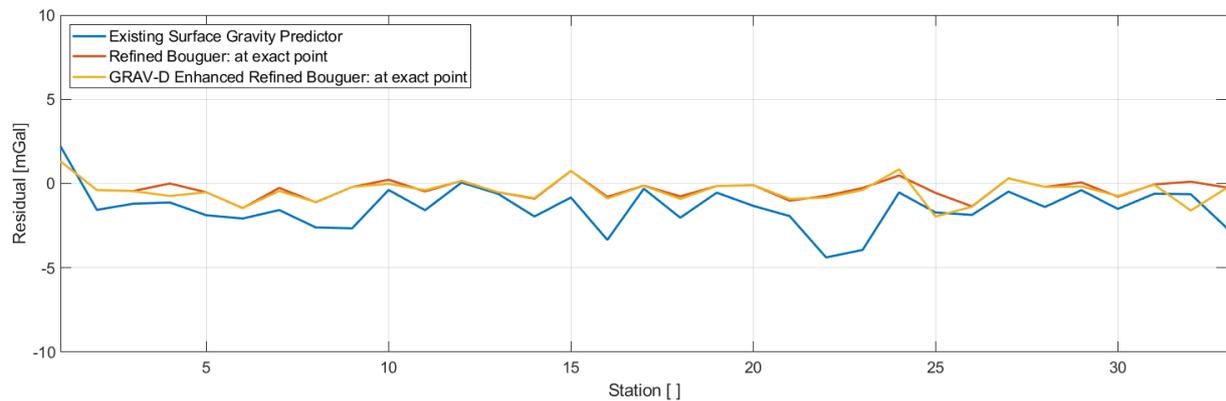


Figure 15: GRAV-D control station prediction residuals for prediction methods that rely on the existing surface gravity data

5. CONCLUSION

The current NGS Surface Gravity Prediction Tool has two concerns: 1) it needs to be consistent with NAPGD2002 and provide more information to users and 2) the performance could be improved. The results from this tool compared against the external validation datasets are consistently worse than many of the other methods evaluated. Additionally, there is a fairly large bias in most cases and the tool does not even work in some geographic situations.

The reference model enhanced with terrain approach generally performs well for most regions – results are less accurate though compared to other methods. This is likely a reflection of the terrain component capturing only 70-80% of the true high frequency gravity signal. This method does have the advantage that it is better suited for areas where existing surface gravity data is in significant error (5+ mGal) depending on the geographical size of surveys in error. It is encouraging that results obtained with this method consistently show an improvement when using the reference model with GRAV-D airborne data. The models that included ERTM2160 typically outperformed those that used the newer SRTM2gravity model (all comparisons for 3 out of 4 validation datasets).

The refined Bouguer anomaly method that uses least squares collocation and a three-dimensional covariance function consistently produces the most accurate results (0.78 mGal on GSVS11, 1.56 mGal on GSVS14, 1.39 mGal with RELBASEA, and 0.65 mGal with GRAV-D control surveys). The localized selection of the LSC parameters allows this method to be adaptable to different distributions of existing surface gravity data and provide a better overall fit.

The GRAV-D enhanced anomaly method does not provide improved results for the various validation datasets evaluated. There may be certain regions where the existing surface gravity data are improved with this method; however, the use of this method is not justified in terms of performance.

REFERENCES

- Filmer, M. S., Hirt, C., & Featherstone, W. E. (2013). Error sources and data limitations for the prediction of surface gravity: a case study using benchmarks. *Studia Geophysica et Geodaetica*, 57(1), 47-66.
- Forsberg, R. (1987). A new covariance model for inertial gravimetry and gradiometry. *Journal of Geophysical Research: Solid Earth*, 92(B2), 1305-1310.
- Forsberg, R. and C.C. Tscherning (2008). An overview manual for the GRAVSOFTE Geodetic Gravity Field Modelling Programs. 2nd Edition.
- Fury, R.J (1999). National Geodetic Survey (NGS) Gravity Prediction Methodology. https://www.ngs.noaa.gov/TOOLS/Gravity/grav_method.html. Accessed 5 February 2020.
- Gesch, D., Evans, G., Mauck, J., Hutchinson, J., & Carswell Jr, W. J. (2009). The national map—Elevation. US geological survey fact sheet, 3053(4).

- Hardy, R.L., and Gopfert, W.M. (1975). Least squares prediction of gravity anomalies, geoidal undulations, and deflections of the vertical with multiquadric harmonic functions', *Geophysical Research Letters*, 2, no. 10, 423-427.
- Hirt C., M. Kuhn, S.J. Claessens, R. Pail, K. Seitz, T. Gruber (2014). Study of Earth's short-scale gravity field using the high-resolution SRTM topography model, *Computers & Geosciences*, 73, 71-80, doi: 10.1016/j.cageo.2014.09.00.
- Hirt, C., M. Yang, M. Kuhn, B. Bucha, A. Kurzmann and R. Pail (2019). SRTM2gravity: an ultra-high resolution global model of gravimetric terrain corrections, *Geophysical Research Letters* 46, doi: 10.1029/2019GL082521.
- Jekeli, C. (1994). Hardy's multiquadric-biharmonic method for gravity field predictions. *Computers & Mathematics with Applications*, 28(7), 43-46.
- Li, X., K. Ahlgren, R. Hardy, J. Krcmaric, Y.M. Wang (2019). The Development and Evaluation of the Experimental Gravimetric Geoid Model 2019. https://beta.ngs.noaa.gov/GEOID/xGEOID19/xGeoid19_tech_details.v10.pdf. Accessed 5 February 2020.
- Moritz, H (1980) *Advanced Physical Geodesy*. Herbert Wichmann, Karlsruhe.
- Priovolos, G. (1988), *Gravity Field Approximation Using the Predictors of Bjerhammar and Hardy*, Report No 387, Dept. of Geodetic Science and Surveying, Ohio State University.
- Smith, D. A., Holmes, S. A., Li, X., Guillaume, S., Wang, Y. M., Bürki, B., ... & Damiani, T. M. (2013). Confirming regional 1 cm differential geoid accuracy from airborne gravimetry: the Geoid Slope Validation Survey of 2011. *Journal of geodesy*, 87(10-12), 885-907.
- Wang, Y. M., Becker, C., Mader, G., Martin, D., Li, X., Jiang, T., ... & Bürki, B. (2017). The Geoid Slope Validation Survey 2014 and GRAV-D airborne gravity enhanced geoid comparison results in Iowa. *Journal of Geodesy*, 91(10), 1261-1276.

BIOGRAPHICAL NOTES

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