

Accuracy of GNSS Observations from Three Real-Time Networks in Maryland, USA

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Key words: Real-time kinematic, smart cities, positioning, global navigation satellite systems, virtual reference system, master-auxiliary concept

SUMMARY

Real-time networks (RTNs) are popular for numerous types of Global Navigation Satellite System (GNSS) surveys because highly accurate geometric coordinates can be derived in seconds to minutes. Due to their accuracy and efficiency, many smart cities employ RTNs for positioning and navigation. Numerous regional, national, and international RTNs are currently available for use. For example, in Maryland of the United States, three independent RTNs are available: (1) Trimble KeyNetGPS and (2) Topcon TopNET live which both employ a virtual reference system; and (3) Leica SmartNet which uses a master-auxiliary concept. To evaluate the accuracy of these RTNs, the latest rover models from each of these three vendors were obtained and connected to the corresponding RTN. Then, over ten days in 2018, a total of 486 independent network real-time kinematic (NRTK) observations of five minutes in duration were collected on nine bench marks distributed across a 4,000 square km area using the three different RTN setups. All three rovers collected both Global Positioning System (GPS) and Globalnaya Navigazionnaya Sputnikovaya Sistema (GLONASS) observables. Observations were taken by equally alternating the rovers during each visit to a mark, and repeat visits to each mark were made at different times each day. Afterwards, the resulting coordinates were differenced with adjusted coordinates derived at each bench mark from a high-accuracy, lengthy static GNSS survey campaign. The coordinate differences were similar in magnitude from each of the three RTNs, indicating that each RTN performed alike in terms of accuracy. The root-mean-square error (RMSE) of the coordinate differences was 2.3 cm horizontally and 4.5 cm in ellipsoid height at 95% confidence. The repetitive NRTK observations were also precise, with 95% of the differences between ± 2.4 cm horizontally and ± 3.4 cm vertically (ellipsoid height). Such positioning errors could be further reduced by construction and adjustment of a survey network of repeat NRTK vectors obtained at each mark. Five different survey networks were developed from the data, consisting of two to six randomly selected, repeat NRTK vectors to each bench mark. Prior to least squares adjustment of each network, the variance-covariance matrices of the NRTK vectors needed to be scaled by variance component estimation procedures to produce realistic observational error estimates for the stochastic modeling. The average scale factor differed for the vectors from each RTN, equal to 1.4 for KeyNetGPS, 14.5 for TopNET live, and 2.2 for Leica SmartNet. When adjusting four repeat NRTK vectors per bench mark, the estimated error by formal error propagation in the adjusted coordinates was less than 1 cm horizontally and 2 cm vertically at 95% confidence at all nine bench marks in the survey network.

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1. INTRODUCTION

Real-time networks (RTNs) are widely used because highly accurate positions can be derived quickly using Global Navigation Satellite System (GNSS) technology. RTNs are commonly used by surveyors, engineers, and other geospatial professionals for rapidly finding geodetic coordinates (i.e., latitude, longitude, and ellipsoid height) on points. Due to their high efficiency and accuracy, RTNs are also popular for navigation, machine control, precision agriculture, structural health monitoring, mobile mapping, and construction. Several regional, national, and international RTNs are available for use.

Because of the advantages of RTNs, some people desire to use this technology for establishing geodetic coordinates on survey control marks. Most traditional guidelines or standards for establishing high-accuracy geodetic control require long-duration, static GNSS observations that must be post-processed and evaluated in the office (e.g., Zilkoski et al. 1997). Unlike static GNSS surveying, RTNs yield real-time baseline solutions from just a few seconds or minutes of a GNSS occupation. Since the baselines are processed in real-time, observations can be evaluated while in the field, and more observations can be collected if repeat check measurements are imprecise. Several studies have been completed showing that geodetic coordinates with horizontal and vertical (ellipsoid height) root-mean-square error (RMSE) less than roughly 3 and 5 cm, respectively, can be obtained in less than 5 minutes (Edwards et al. 2010; Wang et al. 2010; Janssen ad Haasdyk 2011; Martin and McGovern 2012; Smith et al. 2014; Allahyari et al. 2018).

However, RTNs do have some important limitations. First, any error in the coordinates of the RTN base stations, as assigned by the RTN network manager, are propagated to the user. As an example, the National Geodetic Survey (NGS) provides the framework for all positioning activities in the United States. NGS manages a network of continuously operating reference stations (CORSSs), and the CORS Network serves as the backbone of this framework. Thus, coordinates that are assigned for the RTN base stations should be accurately aligned with the CORS Network, but this alignment may not always occur. Second, real-time solutions are more prone to multipathing errors than static-derived solutions, and users should be cautious when selecting sites for setting control. Third, to minimize distance-dependent errors, GNSS baselines from an RTN should be kept under 40 km (Allahyari et al. 2018). As a result, RTNs are typically designed so that the maximum interstation distance of the base stations is under 70 km. Greater interstation distances have been shown to yield less-accurate results (Wang et al. 2010).

Although there is evidence that highly accurate coordinates can be derived with an RTN, it seems that the spacing and geometry of the base stations influences results. In addition, few

comparative studies have been completed between RTNs developed by differing manufacturers. If national specifications are to be developed to allow establishment of geodetic survey control with an RTN, then such specifications should require an RTN to be constructed to meet certain standards, and/or the specifications should be sufficiently conservative so that all suitable RTNs, regardless of the manufacturer, could be used.

Therefore, this paper presents a comparative evaluation on the accuracy of three independent RTNs constructed with differing hardware and software. Within Maryland of the United States, three RTNs were tested: (1) Trimble KeyNetGPS and (2) Topcon TopNET live which both employ a virtual reference system (VRS); and (3) Leica SmartNet which uses a master-auxiliary concept (MAC). The latest rovers from each of these three manufacturers were obtained for the experiment, and each rover was connected to its corresponding RTN. Then, as much as practical, the same GNSS data were collected at nine bench marks distributed over a 4,000 square km area while alternating the rover and the RTN. Hundreds of fixed, network real-time kinematic (NRTK) observations were made on the nine bench marks using the same session duration (5 minutes), satellite signals (GPS and GLONASS), and time of day over a period of ten field days.

The results were evaluated by: (1) differencing the NRTK coordinates with high-accuracy coordinates from a traditional, static, campaign-style GNSS survey; (2) evaluating the repetitiveness or precision of the repeat NRTK observations for each baseline; and (3) estimating error by formal error propagation theory after developing and adjusting a “hybrid survey network” consisting of multiple NRTK vectors to each bench mark.

2. BACKGROUND ON REAL-TIME NETWORKS

Conventional real-time kinematic (RTK) technology uses a single base station that transmits its known coordinates and GNSS observables to the rover receiver, enabling real-time processing of the baseline. However, such configuration generally limits baseline lengths to less than 20 km because, at greater lengths, broadcast orbits and atmospheric delay errors do not sufficiently cancel by differencing GNSS observables collected at the base and rover.

An RTN improves upon these limitations by using a network of continuous GNSS base stations. Atmospheric delay is interpolated between the multiple bases, and the network is also often setup to broadcast ultrarapid ephemerides to reduce satellite orbit error (Zhang et al. 2006; Janssen 2009). Typical practice is to space the base stations every 70 km or less, enabling the real-time computation of baselines up to roughly 40 km in length. Both the bases in the RTN and the rover send data to a centralized network server, and the rover receives data from this server using wireless communication. RTN software uses the data to generate NRTK corrections by fixing integer ambiguities of double-differenced GNSS phase observables. Using the corrections and the assigned geodetic coordinates for the bases, the rover can compute its coordinates in real-time.

Although multiple methods are available, two of the most popular methods for producing NRTK solutions is via VRS and MAC. In the VRS method, the rover transmits a point position

to the server, and the server then assigns this position as the location of an imaginary base station and interpolates network corrections to this virtual location. Pseudo-observables are then generated and transmitted from the virtual base to the rover to be processed using conventional single-base RTK algorithms to solve for geocentric, Earth-centered, Earth-fixed (ECEF) coordinates at the rover. The result is a short, 1 to 3 m long, delta ECEF vector from the VRS to the rover. However, as explained in Weaver et al. (2018), some manufacturers, such as Trimble, also store the ECEF coordinates of the nearest base station in the network. For Trimble, this record is called the "physical reference station" or PRS. Software can be set to transfer the tail of the GNSS vector from the ECEF coordinates of the imaginary VRS to the PRS, thereby producing a delta ECEF vector and its variance and covariance values (i.e., a 3 by 3 variance-covariance matrix) from the PRS to the rover.

In the MAC method, the rover transmits a point position to the server, and then the server assigns a cell of RTN base stations for generating corrections. Usually, the nearest base station is also assigned as the master station, and all other bases in the cell serve as auxiliary stations. Corrections between the auxiliary stations and master base station are computed and transmitted to the rover. The rover then computes a double-differenced solution based on the transmission, and it produces a delta ECEF vector and its variance and covariance values from the master base station to the rover. Thus, similar to the PRS solution mentioned previously, a vector is generated from a physical base station to the rover.

Despite the differences in the VRS and MAC methods, several studies have found that both methods produce coordinates at the rover with similar accuracies (e.g., Edwards et al. 2010; Wang et al. 2010; Martin and McGovern 2012; Allahyari et al. 2018). Interestingly, previous research has also shown that the accuracy in the coordinates hardly improves when increasing the duration of the observational session from 6 s to 600 s (Janssen and Haasdyk 2011; Smith et al. 2014). Allahyari et al. (2018) tested data collected on 38 bench marks using two different RTNs and found only subtle improvement in accuracy when session durations increased from 5 s to 900 s; moreover, the improvement was negligible after 180 s to 300 s (3 to 5 min). Allahyari et al. (2018) also showed that NRTK observations using both GPS and GLONASS observables were more accurate than observations based solely on only using GPS.

Weaver et al. (2018) proposed an efficient, campaign-style survey method wherein repeat NRTK vectors to each bench mark are combined in a survey network for least squares adjustment. After adjustment of thirty so-called "hybrid survey" networks, Weaver et al. (2018) found that ellipsoid height accuracies from formal error propagation were less than 2 cm (95% confidence) when adjusting six or more NRTK observations per mark. The test networks were based on data from only two RTNs. This hybrid survey network method will be further tested and presented herein using survey data collected for this study.

3. METHODS

3.1. Study Area

To evaluate the accuracy of the three RTNs, GNSS observations were made on nine existing bench (passive) marks in Maryland spaced roughly every 30 km. Each bench mark was either

a brass disk set in concrete or was a steel rod driven to refusal. All nine marks were at sites considered suitable for collecting GNSS data. In addition to showing the location and identifier for the nine bench marks, Figure 1 also shows the position of the RTN base stations utilized as either a PRS or as a master base station during the course of the NRTK survey.

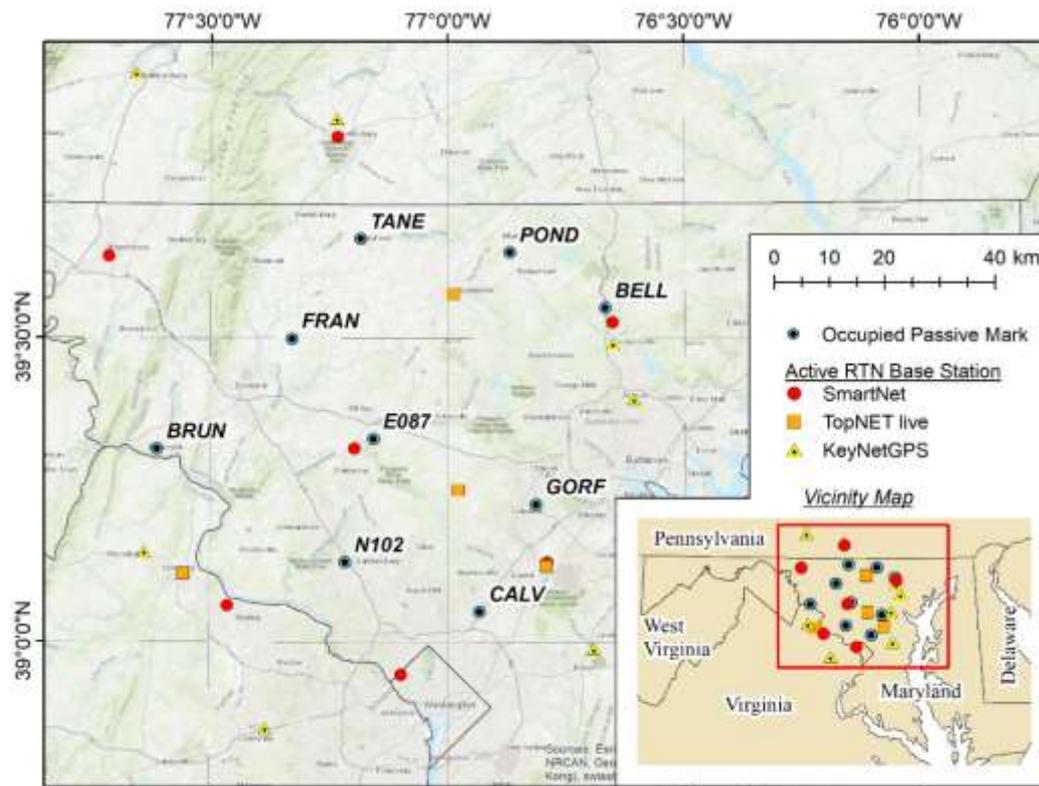


Figure 1. Map of the nine bench marks and active GNSS stations in the RTNs

3.2. Static Survey Campaign, Post-Processing and Adjustment

First, a traditional, campaign-style static GNSS survey was conducted in order to derive high-accuracy geodetic coordinates on the nine bench marks. These coordinates were later used as a basis for evaluating the accuracy of the NRTK observations.

All nine bench marks were observed for two to four independent, 24-h duration static GNSS sessions over five field days in the fall of 2017. During each session, each mark was occupied with a TRM41249.00 NONE antenna attached to a calibrated, fixed-height tripod, and static GPS data were collected. The GPS data were later uploaded to NGS software, *OPUS-Projects*. In addition, 24-h static GPS data collected at eight CORSs were added to each session for post-processing. Afterwards, all of the static GPS data were post-processed and adjusted in *OPUS-Projects* in the same manner as the OP + ADJUST Hub Network design described in detail in Gillins and Eddy (2017). After least squares adjustment of the survey network wherein the published coordinates and standard deviations of the CORSs were held as stochastic constraints,

geodetic coordinates were estimated at the nine bench marks with network accuracies less than 0.5 cm horizontally and 0.6 cm in ellipsoid height (95% confidence).

3.3. NRTK Survey Campaign

Three different, integrated GNSS antenna/receivers were used as rovers in order to collect hundreds of NRTK observations on the nine bench marks (Figure 2). Each rover used cellular data to transmit and receive data from its corresponding RTN: (1) a Trimble R10 rover connected and receiving VRS solutions from KeyNetGPS; (2) a Topcon Hyper V rover connected and receiving VRS corrections from TopNET live; and (3) a Leica GS18 rover connected and receiving MAC (more specifically, “Leica MAX”) corrections from SmartNet.

In order to spread out the time of the measurements, each mark was then visited a total of six times on three different mornings and on three different afternoons between February 21 and March 7, 2018. During each visit, a calibrated, fixed-height tripod was set up over the mark, and then the following nine steps were repeated three times: (1) attach the Trimble R10 antenna to the tripod and wait for it to initialize or fix integer ambiguities; (2) collect and store in the data collector a multi-epoch (i.e., 300 s or 5 min) NRTK observation using both GPS and GLONASS observables; (3) remove the antenna and invert it so that it loses initialization; (4) attach the Leica GS18 antenna and wait for it to initialize; (5) collect and store another NRTK observation using the same session duration and satellite signals for consistency (5 min, GPS+GLONASS); (6) remove the antenna and invert it; (7) attach the Topcon Hyper V antenna and wait for it to initialize; (8) collect and store a 5 min, GPS+GLONASS NRTK observation; and (9) remove the antenna and invert it. Note a consistent, 5-min session duration was chosen based on the aforementioned study on optimal session lengths in Allahyari et al. (2018).



Figure 2. GNSS rovers and data collectors used in the NRTK survey; from left to right: the Topcon Hyper V, Leica GS18, and Trimble R10 integrated GNSS antenna/receiver

Since these steps were repeated three times on six separate visits using three different rovers, a total of 54 NRTK observations (18 with each of the three rovers) were stored at each of the nine marks. Thus, a grand total of 486 NRTK observations were collected. Due to differences in how the network solutions were generated per manufacturer, the NRTK observations were stored as delta ECEF vectors with a 3 by 3 variance-covariance (VCV) matrix: from the PRS to the rover for the Trimble unit; from the master base station to the rover for the Leica unit; and from the VRS to the rover for the Topcon unit. To make the vectors similar for future

analysis, the authors transferred the starting point (or "tail") of each of the vectors stored in the Topcon unit from the ECEF coordinates of the VRS to the assigned ECEF coordinates of the nearest base station in the TopNET live RTN. Thus, it could be said that a PRS-to-rover vector was generated for each of the NRTK observations with the Topcon unit.

In addition to storing delta ECEF vectors, the ECEF coordinates at the rover or end point of each vector were also stored. In addition, the position-, vertical-, and horizontal- dilution of precision (PDOP, VDOP, and HDOP) values and total number of used satellites were also archived for each NRTK observation as quality control measures.

3.4. Hybrid Survey Networks for Adjustment

The delta ECEF vectors from the NRTK survey were downloaded and prepared into sets of hybrid survey networks for least squares adjustment. Weaver et al. (2018) provides a flow chart, diagram, and detailed discussion on the development a hybrid survey network; therefore, only a brief summary is given here. In this method, repeat NRTK vectors are first selected for use in the network, and the static GNSS data collected at the starting point of each of these vectors (i.e., at the PRSs or master base stations) are downloaded from the website of the RTN. Additionally, multiple CORSs near the survey project are identified for use as control, and the static GNSS data collected at these CORSs during the time of the NRTK survey are downloaded. All of this static GNSS data are post-processed to yield baseline solutions between the RTN base stations (PRSs or master base stations) and the CORSs. Afterwards, the static-derived baseline solutions and the NRTK vectors are combined in the network and adjusted by least squares. The published coordinates of the CORSs are used as constraints.

After downloading the data collectors for the three rovers, 19 different RTN bases (eight in KeyNetGPS, four in TopNET live, and seven in SmartNet) were found to have been utilized as either a PRS or a master base station over the course of the NRTK survey campaign (Figure 1). GPS data collected at each of these base stations during the 10 field days of the NRTK campaign were downloaded from the websites for the RTNs. The GPS data collected at the base stations were prepared into 24-h duration static GPS observation files, and these files were uploaded to *OPUS-Projects* for post-processing. The files were uploaded in three different projects, one for each RTN. Next, 24-h duration GPS data collected on the same ten days at the same eight CORSs in the previously mentioned static survey campaign were also uploaded in all three projects in *OPUS-Projects* for use as control. Then, the baselines were post-processed between the eight CORSs and the RTN base stations.

Lastly, in order to test accuracy versus number of occupations, the NRTK vectors were carefully sampled and grouped by RTN into sets of 2, 3, 4, 5, and 6 repeat vectors to each bench mark. In order for each vector to be considered as an "independent" observation, each vector was sampled so that none of the repeat vectors in the set were collected during the same visit. In other words, the sampling was done, to the extent possible, so that the repeat vectors in each set were collected at different times of day with a different setup of the tripod. For example, for the set of 2 vectors, one NRTK observation from a morning visit and one from an afternoon visit of the bench mark were selected.

The five sample sets of NRTK vectors grouped by three different RTNs yielded 15 networks of vectors for testing. The static-derived baseline solutions from *OPUS-Projects* were then combined with each of these networks of NRTK vectors, corresponding according to RTN, yielding 15 hybrid survey networks for adjustment. Figure 3 presents an example of one of these hybrid survey networks using the vectors and data from SmartNet. Minimally and fully constrained least squares adjustments were performed for each network.

4. RESULTS AND DISCUSSION

4.1. Number of Satellite Vehicles, Dilution of Precision for NRTK Observations

Figure 4 shows histograms of the total number of satellite vehicles used, PDOP, HDOP, and VDOP for the 486 NRTK observations. As shown, collecting data from both GPS and GLONASS vehicles resulted in the use of 10 to 18 satellites (median 15) per NRTK observation. The use of so many satellites at sites with good overhead visibility produced observations with excellent (low) dilution of precision (DOP) values. The median PDOP, VDOP, and HDOP values for all of the observations were 1.49, 0.74, and 1.30, respectively. The DOP values tended to be smaller when using SmartNet, but all values were much smaller than 7, indicating high quality in the NRTK observations.

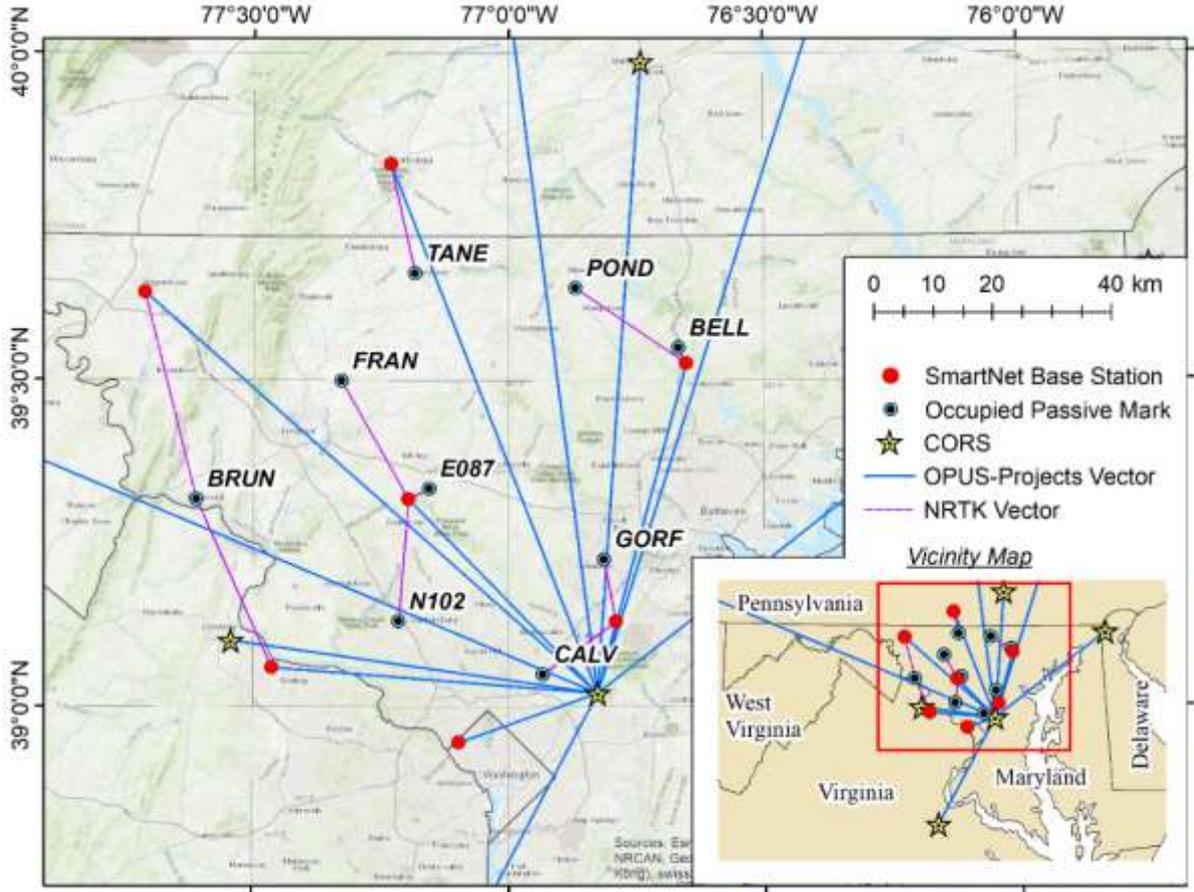


Figure 3. Example hybrid survey network using NRTK vectors and data from SmartNet

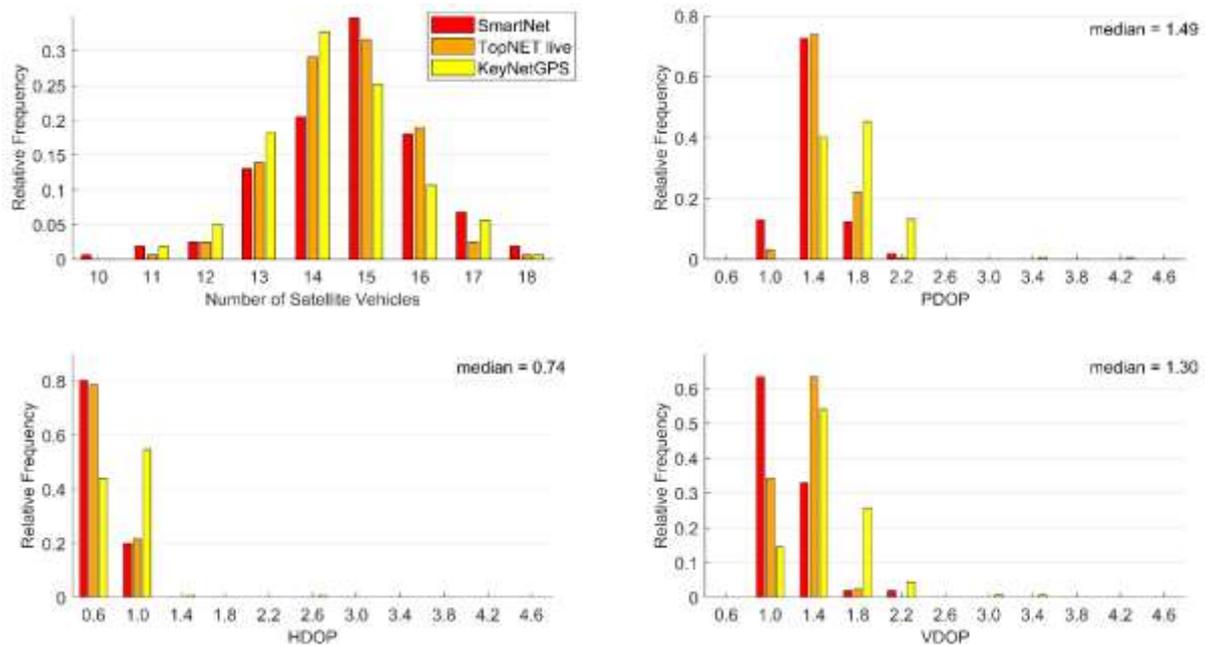


Figure 4. Histograms of the total number of satellite vehicles used, PDOP, HDOP, and VDOP for the NRTK observations

4.2. Coordinates from NRTK Observations Versus from Static Survey

For each mark, the coordinates derived at the rover in real-time from the 5-min GPS+GLONASS NRTK observations were differenced with the adjusted coordinates from the static survey campaign. Figure 5 displays vertical (ellipsoid height) differences at each mark for their 54 NRTK observations (18 per each of three rovers). Table 1 summarizes the statistics for these horizontal and vertical coordinate differences, pooled according to RTN.

Table 1. Summary statistics of the horizontal and vertical (ellipsoid height) coordinate differences between the NRTK observations and the adjusted static survey

| | Horizontal Difference (cm) | | | Vertical Difference (cm) | | |
|----------|----------------------------|-----------|----------|--------------------------|-----------|----------|
| | TopNET live | KeyNetGPS | SmartNet | TopNET live | KeyNetGPS | SmartNet |
| minimum | 0.1 | 0.1 | 0.1 | -4.8 | -6.0 | -4.5 |
| maximum | 2.8 | 3.5 | 1.8 | 6.6 | 6.4 | 6.0 |
| mean | 0.9 | 1.1 | 0.7 | 0.6 | 0.7 | 0.6 |
| st. dev. | 0.5 | 0.7 | 0.4 | 2.8 | 2.2 | 1.9 |
| RMSE | 1.0 | 1.4 | 0.8 | 2.9 | 2.2 | 2.0 |

Some of the error in the coordinates could be simply due to errors in the coordinates assigned to the base stations by the RTN managers. Errors in the coordinates of the bases will propagate to the coordinates derived at the rover. To investigate, only the aforementioned baseline solutions from the static GPS post-processing in *OPUS-Projects* between the base stations and CORSs were adjusted by least squares. Again, the published coordinates and their standard

deviations for the same eight CORSs were used as stochastic constraints in the adjustment. The resulting adjusted coordinates for the base stations were then differenced with the assigned coordinates by the RTN managers (Table 2). The coordinates generally differed less than 0.5 cm horizontally and 1 cm vertically at the 19 base stations; however, these differences are large enough to influence some of the statistics presented in Table 1. Interestingly, there appears to be a -1.0 cm bias in the assigned ellipsoid heights for the eight base stations in KeyNetGPS.

Table 2. Summary statistics of the horizontal and vertical coordinate differences between the adjusted coordinates from the post-processing of the static GPS data collected at the base stations in *OPUS-Projects* and the coordinates assigned by the RTN managers

| | Horizontal Difference (cm) | | | Vertical Difference (cm) | | |
|----------|----------------------------|-----------|----------|--------------------------|-----------|----------|
| | TopNET live | KeyNetGPS | SmartNet | TopNET live | KeyNetGPS | SmartNet |
| minimum | 0.1 | 0.1 | 0.4 | -1.0 | -2.8 | -1.0 |
| maximum | 0.8 | 1.0 | 0.7 | 0.7 | -0.3 | 1.5 |
| mean | 0.4 | 0.5 | 0.5 | -0.2 | -1.0 | 0.5 |
| st. dev. | 0.3 | 0.3 | 0.1 | 0.7 | 0.8 | 0.9 |
| RMSE | 0.5 | 0.6 | 0.6 | 0.6 | 1.3 | 0.9 |

The starting point for all of the collected NRTK vectors in the real-time survey were updated with the newly adjusted coordinates computed at the base stations, and new coordinates were then output and the ending point of the vector (at the rover). At each bench mark, these updated coordinates were again differenced with the coordinates from the traditional static survey campaign (Table 3). Comparing Tables 1 and 3, the RMSE values were reduced by 10% except for the vertical RMSE for SmartNet. Both the horizontal and vertical RMSE for SmartNet were slightly lower than for the other two networks, but all three networks performed similarly.

Table 3. Summary statistics of the horizontal and vertical (ellipsoid height) coordinate differences between the NRTK observations after correcting the coordinates of the RTN base stations and the adjusted static survey campaign

| | Horizontal Difference (cm) | | | Vertical Difference (cm) | | |
|----------|----------------------------|-----------|----------|--------------------------|-----------|----------|
| | TopNET live | KeyNetGPS | SmartNet | TopNET live | KeyNetGPS | SmartNet |
| minimum | 0.1 | 0.1 | 0.0 | -4.4 | -6.3 | -3.1 |
| maximum | 2.7 | 3.2 | 1.6 | 6.1 | 5.9 | 7.3 |
| mean | 0.8 | 1.0 | 0.6 | 0.3 | -0.4 | 1.4 |
| st. dev. | 0.4 | 0.6 | 0.3 | 2.4 | 2.2 | 1.7 |
| RMSE | 0.9 | 1.2 | 0.7 | 2.4 | 2.2 | 2.2 |

Since the statistics for all three RTNs are similar, by pooling the data together, the horizontal and vertical RMSE of the coordinate differences is 0.9 and 2.3 cm, respectively. These values can approximate accuracy at 95% confidence by multiplying them by 2.45 (bivariate) and 1.96 (univariate distribution), resulting in horizontal and vertical scaled RMSE values of 2.3 and 4.5 cm, respectively.

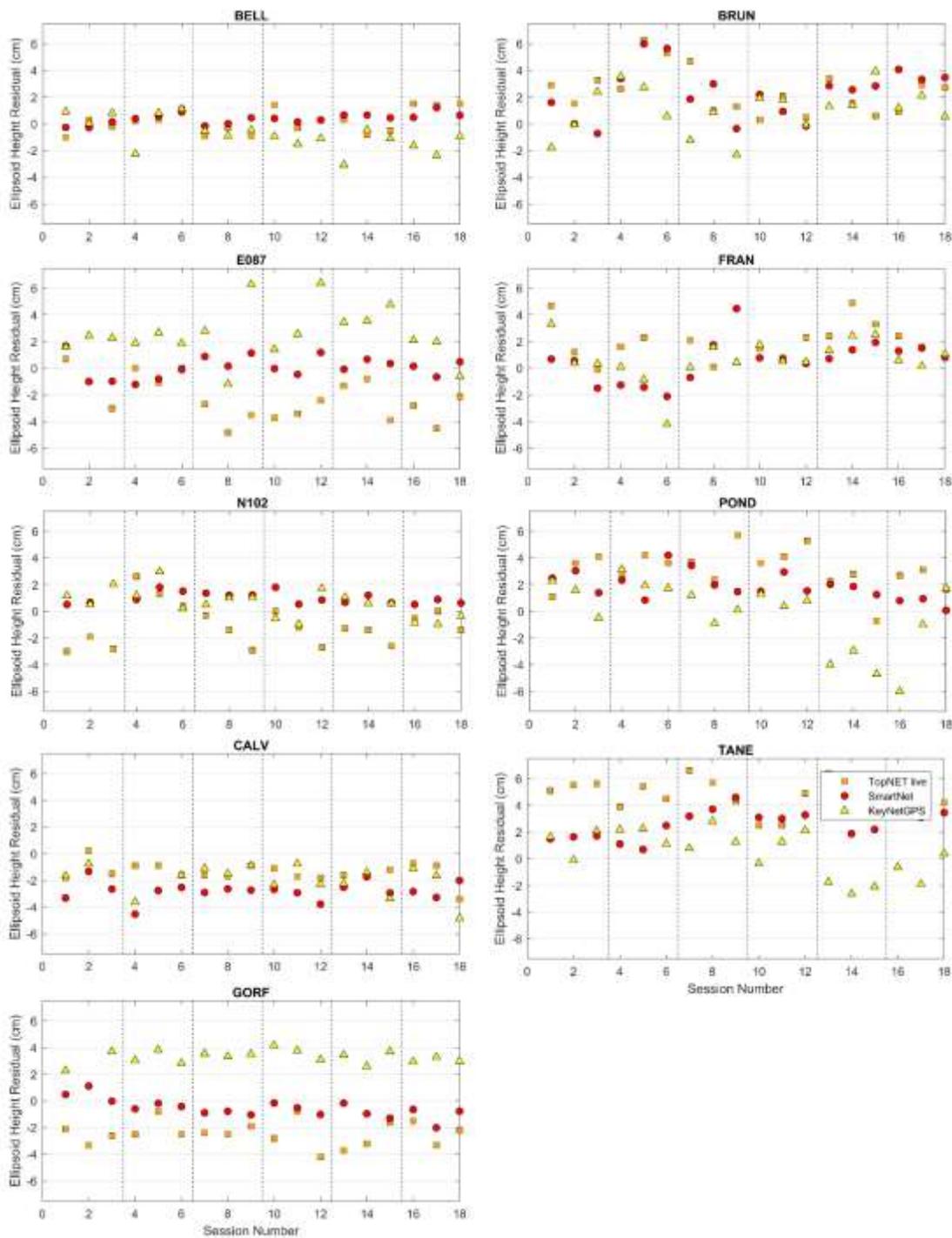


Figure 5. Ellipsoid heights from the NRTK observations minus the adjusted ellipsoid height from the traditional static GNSS survey for each of the nine bench marks

4.3. Precision of Repeat NRTK Observations

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In the previous section, the coordinates from the more-traditional static survey campaign were used as "true" coordinates or as a basis for evaluating the accuracy of the NRTK observations. However, these coordinate have some error, as noted in Section 3.2 of this paper. Another method for evaluating the NRTK data is to examine the precision of repetitive NRTK observations. Repeat vectors are not influenced by errors in the coordinates of the base stations or in the "true" coordinates.

All unique baselines from an RTN base station to the rover from the NRTK survey were identified. Up to six repeat vectors per baseline were then considered truly independent because they were collected with a unique setup of the tripod during a visit to the mark. The up to six independent vectors per baseline were differenced with one another.

Figure 6 provides histograms of the repeat vector differences in delta northing, easting, and up. The RMSE values in delta northing, easting, and up were 0.7, 0.7, and 1.7 cm, respectively. Nearly all of the differences are less than 2 cm horizontally and 5 cm vertically.

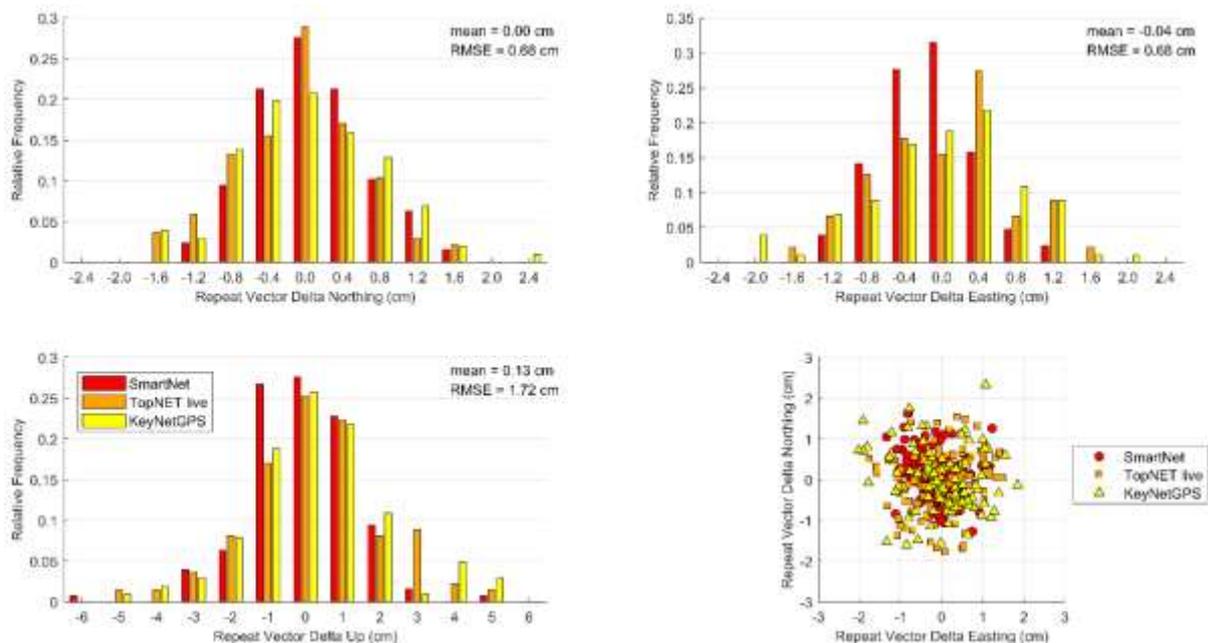


Figure 6. Histograms of the differences in northing, easting, and up for the repeat NRTK vectors for each baseline

4.4. Formal Error Propagation Estimates from Hybrid Survey Networks

The final method for evaluating the accuracy of the NRTK observations involved developing and adjusting hybrid survey networks. This approach allows estimation of the accuracy of the adjusted coordinates by formal error propagation theory. As mentioned, 15 hybrid survey networks were developed using 2, 3, 4, 5, or 6 independent repeat NRTK vectors to the bench marks from each of the three RTNs.

As discussed in Weaver et al. (2018), only the NRTK vectors for each of the 15 networks were first minimally constrained and adjusted in order to solve for variance components for weighting the VCV matrices of the static and NRTK vectors. This step is crucial because VCV matrices from different GNSS surveying methods (e.g., static versus kinematic) and baseline-processing software are incompatible. The adjustments were done in NGS software *ADJUST* which iterates and solves for a separate horizontal and vertical standard deviation (i.e., square-root of the variance) component. Table 4 presents these components for all 15 test networks. As shown, the values are generally greater than 1, indicating over-optimism in the estimated variances for the NRTK vectors; note the NRTK vectors from TopNET live have very optimistic variance estimates. The VCV matrices for the NRTK vectors in each of the networks were multiplied by the factors shown in Table 4. The same procedure was done for scaling the VCV matrices of the baseline solutions from the static GPS data processed in *OPUS-Projects*.

Afterwards, the published coordinates and standard deviations of the eight CORSs were used as constraints, and all 15 hybrid survey networks were adjusted in *ADJUST*. The variances of unit weight for all 15 networks were close to 1 and passed the χ^2 statistical hypothesis test at the 95% confidence level, indicative that the stochastic models for the adjustments were valid.

Figure 7 presents the mean and spread in the estimated horizontal and vertical network accuracy of the adjusted coordinates for all nine bench marks in the 15 hybrid survey networks. Note "network accuracy" is defined in FGDC (1998) as the estimated standard deviation of the adjusted coordinate, scaled to 95% confidence. As should be expected, as the number of NRTK vectors to each mark increases, the accuracy improves. Figure 7 is useful for deciding how many times a mark must be independently occupied in order to achieve a particular network accuracy. For example, if the mark is only observed with three NRTK observations, horizontal network accuracies may range from 0.4 to 1.4 cm and vertical network accuracies may range from 0.7 to 2.7 cm (95% confidence). When four repeat NRTK observations were made at every bench mark in the network, horizontal and vertical network accuracies were all less than 1 cm and 2 cm, respectively.

Table 4. Standard deviation components for scaling the estimated VCV matrices of the NRTK vectors in the hybrid survey test networks

| Network | SmartNet | | KeyNetGPS | | TopNET live | |
|-----------|-------------------|-----------------|-------------------|-----------------|-------------------|-----------------|
| | Horizontal Factor | Vertical Factor | Horizontal Factor | Vertical Factor | Horizontal Factor | Vertical Factor |
| 2 Vectors | 2.345 | 2.326 | 1.288 | 0.669 | 15.123 | 15.000 |
| 3 Vectors | 2.288 | 2.318 | 1.604 | 1.626 | 13.767 | 15.130 |
| 4 Vectors | 2.123 | 2.268 | 1.171 | 1.251 | 8.681 | 14.731 |
| 5 Vectors | 2.375 | 2.229 | 1.532 | 1.357 | 14.467 | 16.181 |
| 6 Vectors | 2.202 | 2.116 | 1.501 | 1.343 | 14.104 | 16.441 |

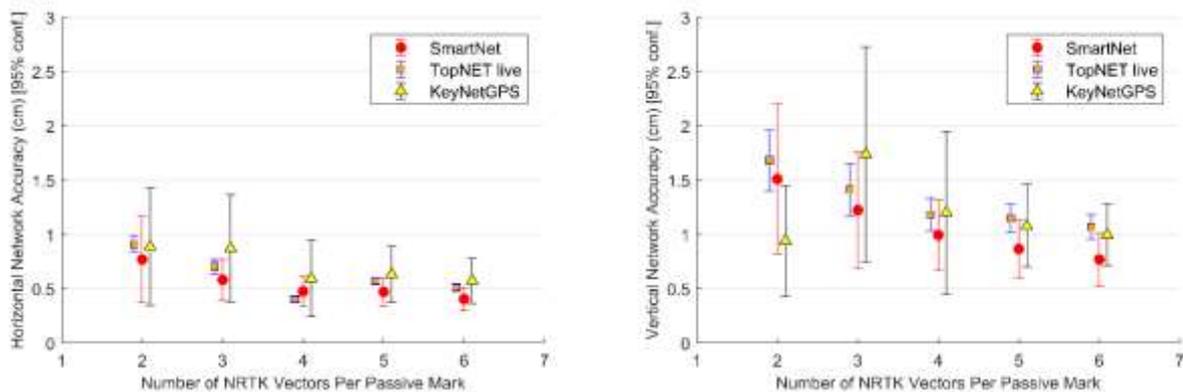


Figure 7. Estimated horizontal and vertical network accuracy at the bench marks versus number of independent NRTK vectors to each bench mark

5. CONCLUSIONS

A total of 486, 5-min duration, GPS+GLONASS NRTK observations were collected on nine bench marks distributed over a 4,000 square km area with rovers connected to three different RTNs in Maryland. Each RTN was developed with equipment and software from a different manufacturer, yet all three RTNs performed similarly in terms of accuracy. When differenced with coordinates from a static GNSS survey campaign, the horizontal and vertical RMSE of the NRTK-derived coordinates was 2.3 cm horizontally and 4.5 cm vertically at 95% confidence. Repetitive NRTK vectors on each baseline differed between ± 2.4 cm horizontally and ± 3.4 cm vertically at 95% confidence. As a final accuracy evaluation, hybrid survey networks consisting of repeat NRTK vectors and baseline solutions from post-processing static GPS data collected at RTN base stations and CORSs were adjusted by least squares. Prior to adjustment, the VCV matrices of the vectors were scaled by variance-component estimation. Adjustment of hybrid survey networks with four repeat NRTK vectors per bench mark produced network accuracies at 95% confidence for the adjusted coordinates at all bench marks less than 1 cm horizontally and 2 cm vertically (ellipsoid height). In addition to the benefits of using efficient and accurate NRTK vectors, the hybrid survey network approach makes use of redundant vectors for checking data and avoiding blunders. The approach also provides traceability because the NRTK vectors are tied to an RTN base station which is tied to CORSs. Finally, these networks ensure the survey is referenced to the published coordinates of the CORSs which are held as constraints in the adjustment.

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BIOGRAPHICAL NOTES

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