Terrestrial Laserscanning - Modeling of Correlations and Point Clouds for Deformation Analysis

Corinna Harmening, Stephanie Kauker, Hans Neuner, Volker Schwieger

78th FIG Working Week 2016

5 May 2016

Outline

1. Motivation
2. Modelling of correlations
3. Modelling of the areal trend component
4. Conclusion

Motivation

Fig.1: Scanning HFT Stuttgart

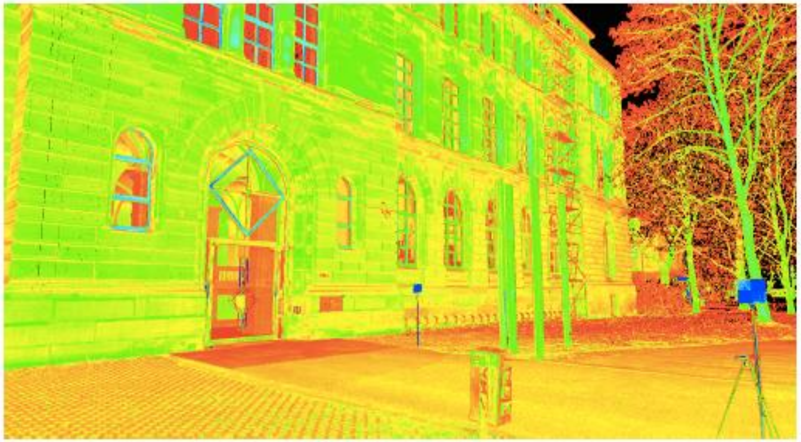

Fig.2: Point cloud of HFT scan

- **Goal**: detecting deformation on any point on an object surface
- **Observations**: horizontal rotation angle, vertical rotation angle, slope distance
- Substantial error sources cause **stochastical relations** within a point cloud

Motivation (cont'd)

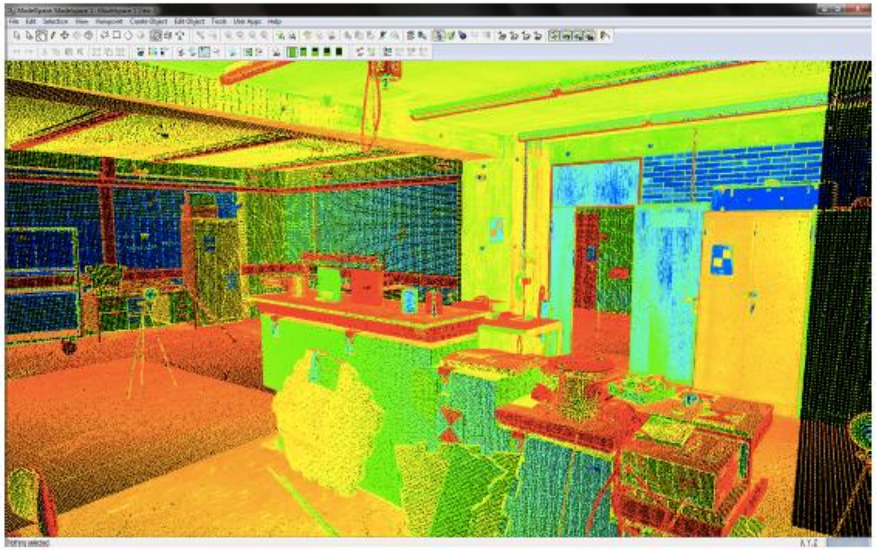

Fig.3: Point cloud of test room, US

Fig.4: Leica HDS 7000

- Detection by high-resolution data of terrestrial laser scanners
  - ➤ **Highly correlated** data
- Reproducing points from epoch to epoch
  - ➤ Development of **non-linear spatio-temporal** method for modelling a time-varying object surface

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Synthetic covariance matrix

- **Multidimensional** measured values and random deviations
  - **Classification** of elementary errors
  - Requirement of **influencing** and **covariance matrices**

    1. **Non-correlating** errors: $\boldsymbol{D_k}$, $\boldsymbol{\Sigma_{\delta\delta,k}}$
    2. **Functional correlating** errors: $\boldsymbol{F}$, $\boldsymbol{\Sigma_{\xi\xi}}$
    3. **Stochastic correlating** errors: $\boldsymbol{G_h}$, $\boldsymbol{\Sigma_{\gamma\gamma,h}}$

- **Synthetic covariance matrix**

$$\boldsymbol{\Sigma}_{ll} = \underbrace{\sum_{k=1}^{p} \boldsymbol{D}_k \cdot \boldsymbol{\Sigma}_{\delta\delta,k} \cdot \boldsymbol{D}_k^T}_{\text{non-correlating}} + \underbrace{\boldsymbol{F} \cdot \boldsymbol{\Sigma}_{\xi\xi} \cdot \boldsymbol{F}^T}_{\text{functional correlating}} + \underbrace{\sum_{h=1}^{q} \boldsymbol{G}_h \cdot \boldsymbol{\Sigma}_{\gamma\gamma,h} \cdot \boldsymbol{G}_h^T}_{\text{stochastic correlating}}$$

Synthetic covariance matrix (cont'd)

- Non-correlating

$$D_k = \begin{bmatrix} \frac{\partial l_1}{\partial \delta_{1k}} & 0 & \cdots & 0 \\ 0 & \frac{\partial l_2}{\partial \delta_{2k}} & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & \cdots & \frac{\partial l_n}{\partial \delta_{nk}} \end{bmatrix}$$

$$\Sigma_{\delta\delta,k} = \begin{bmatrix} \sigma_{1k}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{2k}^2 & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & \cdots & \sigma_{nk}^2 \end{bmatrix}$$

- Functional correlating

$$F = \begin{bmatrix} \frac{\partial l_1}{\partial \xi_1} & \frac{\partial l_1}{\partial \xi_2} & \cdots & \frac{\partial l_1}{\partial \xi_m} \\ \frac{\partial l_2}{\partial \xi_1} & \frac{\partial l_2}{\partial \xi_2} & \cdots & \frac{\partial l_2}{\partial \xi_m} \\ \vdots & \vdots & & \vdots \\ \frac{\partial l_n}{\partial \xi_1} & \frac{\partial l_n}{\partial \xi_2} & \cdots & \frac{\partial l_n}{\partial \xi_m} \end{bmatrix}$$

$$\Sigma_{\xi\xi} = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & \cdots & \sigma_n^2 \end{bmatrix}$$

- Stochastic correlating

$$G_h = \begin{bmatrix} \frac{\partial l_1}{\partial \gamma_{1h}} & 0 & \cdots & 0 \\ 0 & \frac{\partial l_2}{\partial \gamma_{2h}} & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & \cdots & \frac{\partial l_n}{\partial \gamma_{nh}} \end{bmatrix}$$

$$\Sigma_{\gamma\gamma,h} = \begin{bmatrix} \sigma_{1h}^2 & \sigma_{12h} & \cdots & \sigma_{1nh} \\ \sigma_{12h} & \sigma_{2h}^2 & \cdots & \sigma_{2nh} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1nh} & \sigma_{2nh} & \cdots & \sigma_{nh}^2 \end{bmatrix}$$

Elementary errors	No correlations	No correlations	**Correlations**
Observations	No correlations	**Correlations**	**Correlations**

Elementary errors of terrestrial laser scanners

- **Assumption**: the main error sources of a total station appear for a terrestrial laser scanner
- **Definition** and **classification** of the elementary errors

Correlation	Error	Correlation	Error
Non	Noise *	Functional	Angle of incidence Edges
Stochastic	Penetration depth Roughness Reflectivity Colour		Range finder *
			Zero point *
			Collimation axis *
			Horizontal axis *
	Temperature * Pressure * Partial water vapour pressure *		Vertical collimation *
			Tumbling error *
			Eccentricity of the collimation axis *

* Considered for simulation
Legend: Instrumental, atmospheric, object based

Simulation and results

- Change of range

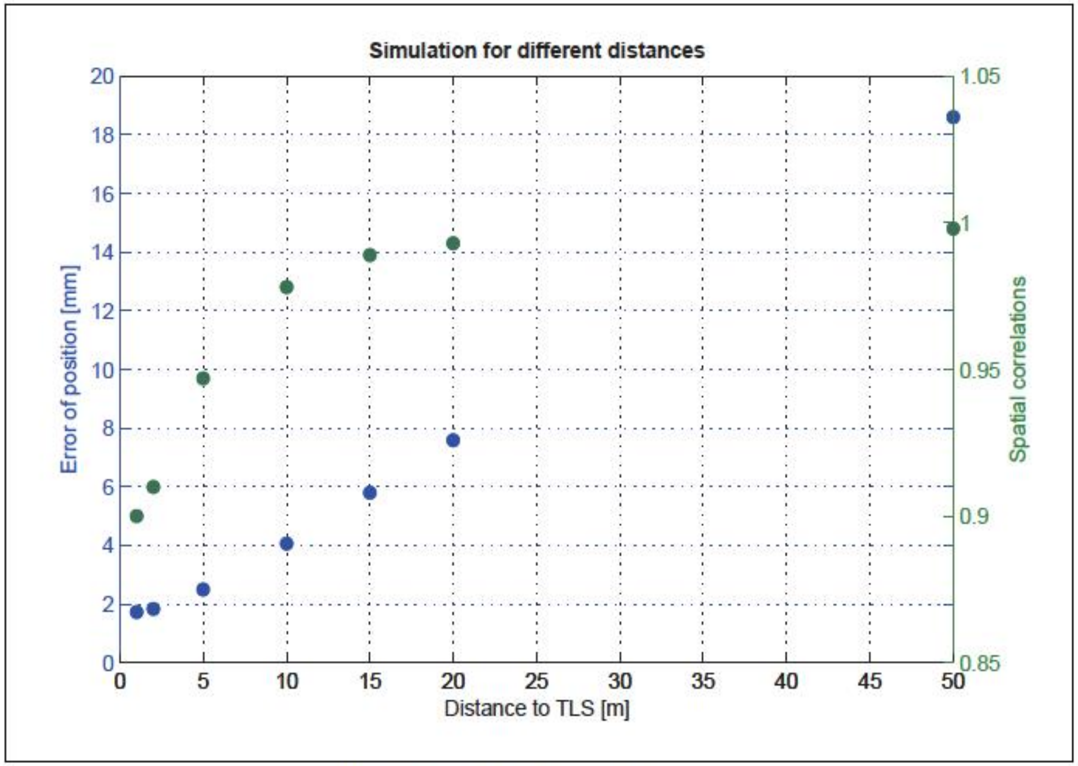

Fig.9: Effect of different distances

➤ Dependency between range and spatial correlations

➤ Rise of distance leads to increasing correlations and worsening of positional error

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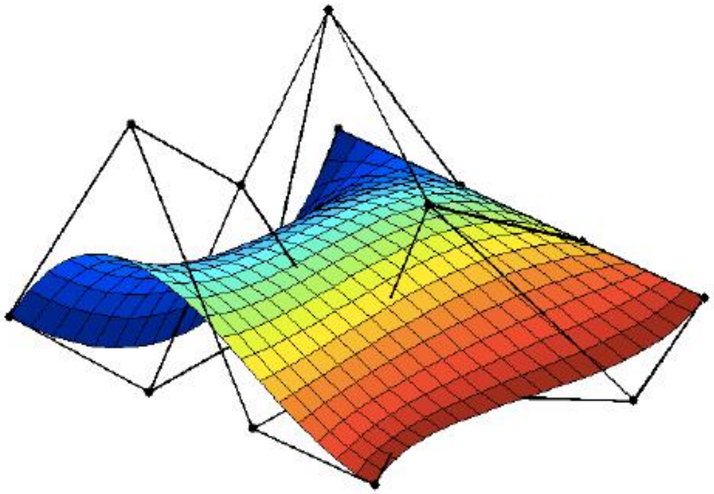
Modelling of point clouds using B-splines



B-spline surface to model point clouds

$$\hat{\mathbf{S}}(u,v) = \mathbf{S}(u,v) + \mathbf{e} = \sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)\mathbf{P}_{ij}$$

$\mathbf{S}(u,v)$: surface point with surface parameters u and v

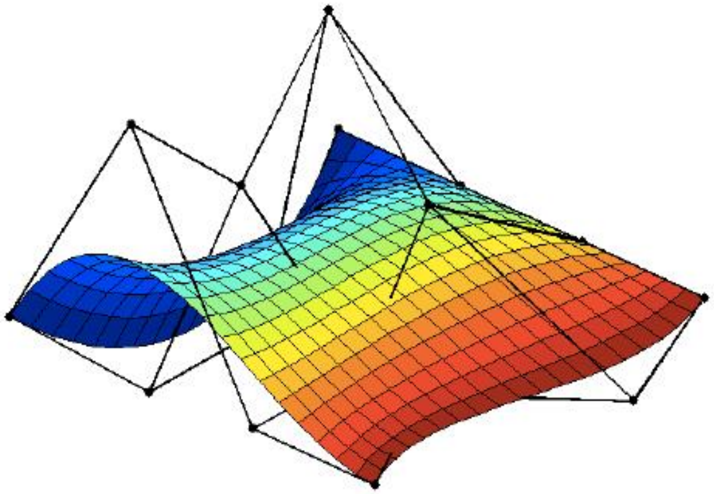
$\mathbf{e}$: residuals

$\mathbf{P}_{ij}$: $(n+1)x(m+1)$ control points

$N_{i,p}(u)$: i-th B-spline basis function of degree p

$N_{j,q}(v)$: j-th B-spline basis function of degree q

Modelling of point clouds using B-splines



B-spline surface to model point clouds

$$\hat{\mathbf{S}}(u,v) \quad = \quad \mathbf{S}(u,v) + \mathbf{e} = \sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) \mathbf{P}_{ij}$$

$\mathbf{S}(u,v)$	:	surface point with surface parameters u and v
$\mathbf{e}$	:	residuals
$\mathbf{P}_{ij}$	:	$(n+1)x(m+1)$ control points
$N_{i,p}(u)$	:	i-th B-spline basis function of degree p
$N_{j,q}(v)$	:	j-th B-spline basis function of degree q

Model selection

- **Basic principles** of model selection
  - Choosing a model out of a set of candidate models
  - Information criteria evaluate each model
  - Based on Maximum Likelihood (ML) Theory

- **Asymptotic efficient** criteria
  - Phenomenon has infinite dimension
  - Best approximation of the true phenomenon
  - Minimizing a distance measure
  - Example: AIC

- **Asymptotic consistent** criteria
  - Phenomenon has finite dimension
  - True dependency is contained in the set of candidate models
  - Identifying the true dependency
  - Example: BIC

Model selection by AIC and BIC

- **Akaike Information Criterion (AIC)**
  - Asymptotic efficient
  - Optimal model minimizes the Kullback-Leibler-distance
  - AIC $= -2\log(\mathcal{L}(\hat{\theta}|y)) + 2K$

- **Bayesian Information Criterion (BIC)**
  - Asymptotic consistent
  - Optimal model is a posteriori most likely
  - BIC $= -2\log(\mathcal{L}(\hat{\theta}|y)) + K\log(l)$

- **AIC** and **BIC**
  - Based on two completely different concepts
  - Penalized log-likelihood criteria
  - Find a balance between approximation quality and complexity
  - BIC's penalty is stronger than AIC's for $l \geq 8$

Choosing the optimal number of control points

- **Procedure**
  - Simulated B-spline surface with $n+1=5$ and $m+1=7$ control points
  - Superimposed by white noise with $\sigma = 2\,mm$
  - Different sample sizes and different realizations
  - Estimation of B-spline surfaces with varying number of control points
  - Evaluation of each estimated curve with AIC and BIC
  - Choosing the **surface with the best score** to be the **optimal** one

Choosing the optimal number of control points (cont'd)

l	AIC	BIC
900	11,**7**	**5**,**7**
900	**5**,11	**5**,**7**
900	11,11	**5**,**7**
3600	**5**,10	**5**,**7**
3600	11,11	**5**,**7**
3600	8,11	7,**7**
4900	7,11	**5**,9
4900	11,11	**5**,10
4900	**5**,9	**5**,9

Evaluation: AIC

- Distinct tendency to overestimate the number of control points
- Only in few cases at least one of the two parameters is determined correctly

➤ **Unsuitable** to determine the number of control points

Evaluation: BIC

- Satisfying results in case of the smaller sample sizes
- Becomes unstable with growing sample size
- At least one of the parameters is identified correctly

➤ **Suitable** to determine the number of control points

Outline

Conclusion

- Conclusion:
  - Computation of synthetic covariance matrix
  - Mean positional error of 2.5 mm and spatial correlations of 0.95 at 5 m distance
  - Spatial correlations depend distance to each other

  - Modelling freeform surfaces by means of B-splines
  - Optimization of the number of control points with AIC and BIC
  - BIC is the more appropriate method regarding the simulated data

- Future work:
  - Integration of object based impacts
  - Evaluation by means of empirical values

  - Identification of additional criteria
  - Adoption of decorrelation techniques

Thank you for your attention

Contact:

- Dipl.-Ing. Stephanie Kauker
  Institute of Engineering Geodesy, University of Stuttgart, Germany
  Mail: stephanie.kauker@ingeo.uni-stuttgart.de

- M. Sc. Corinna Harmening
  Department of Geodesy and Geoinformation,
  Vienna University of Technology, Austria
  Mail: corinna.harmening@geo.tuwien.ac.at