Distance Dependent Approach for the Determination of Standard Land Values by Multiple Regression

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Key words: Standard land values, multiple regression, distance dependent approach, travel time, objectification

SUMMARY

The topic of this paper is the question to what extent may be achieved an objectification of determination of standard land values by a multivariate polynomial which describes price changes by parameters depending on distance. The distance is modeled by travel time. Especially the registration of travel time by public transport demands big effort.

There must be an almost radial structure of decreasing standard land values with increasing distance to the city center for the applicability of the algorithm.

First results of the multiple regression demonstrate the big influence of the distance to the city center, so that further distances to schools or basic services do not lead to a qualitative improvement of the algorithm. In the considered example, the travel time by public transport was even so dominant that all other particular parameters had no significant influence. The influence of infrastructure has to be considered by the committee of valuation experts by means of professional discretion.

The algorithm provides convincing results taking into account the precision of stand land value estimation. The level of standard land values is very good modeled by the multivariate polynomial, if there are high quality standard land values.

Generally, it will not be always possible to adopt the exact result of the calculation, but the algorithm provides very important arguments for the justified raise or reduction of standard land values or to its definition, respectively, if the dispersion of existing buying prices is too big or if there are no buying prices.

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1. INTRODUCTION

In Germany, there are committees of valuation experts in every bigger town or district. These committees are formed by at least three members and are responsible for the provision of important data concerning the real estate market. An important basis for their work is purchasing price data. In order to enable the compilation of purchasing price data, a copy of every contract of transactions on the real estate market has to be sent by the office where this is recorded to the committee. The second step is the analysis of these contracts and the derivation of standard land values and further valuation data. The results have to be published at least every 2 years.

Standard land values are average land values of defined zones. The demarcation of these zones is based on the principle that all plots inside of a zone should have similar characteristics (especially type and degree of building and land use). The standard land value is determined by sales comparison approach, if enough comparable plots are available. Especially in rural areas is oftentimes a lack of available information and standard land values are calculated by means of procedures that are based on subjective estimations of valuation experts. For this reason, it has to be investigated, if there are alternative approaches to obtain an objectification. The topic of this paper is the question, if this objective may be achieved by means of a multivariate polynomial which describes price changes by parameters depending on distance. The investigation is limited to residential areas because this is the prevailing land use in the surroundings of cities and the dependence of land values on the distance to the city centre is much higher than in commercial areas.

The pioneering hedonic price analysis dates back to the first half of the 20th century. One of the first papers was published by Andrew Court in 1939 [Goodman, 1998]. A theoretical foundation was given in the 70th by Sherwin Rosen [Rosen, 1974]. A good overview of the application of hedonic methods in the area of real estate is given by [Sirmans et al., 2006].

The monocentric model of land values was developed by David Ricardo in the 19th century for rural areas. Heinrich von Thünen transferred this knowledge to urban areas. Further fundamental work has been done by Alonso, Mills and Muth in the 70th of the last century. However, from the beginning of the 1980th the empirical relevance of the monocentric model has been questioned. But more recently, research has shown that the monotonically house price gradient has regained relevance [Osland, 2008]. For example, a market analysis in Shanghai from 2008 shows clearly the impact of distance to the city centre on housing prices. In this study was included the distance in kilometre and the availability by public transport, but it was not based on the travel time [Chen and Hao, 2008]. Another recently published paper demonstrates the influence of decreasing travel times due to a new subway line on property values [Hiironen et al., 2015]. However, most of existing studies focus on particular factors or they use the geometric distance as indicator. In this paper is used the travel time to include information concerning the quality of the road network and the public transport.

2. MATHEMATICAL MODELL AND ACCURACY MEASURES

The pricing multivariate polynomial has the following basic structure:

$$BW = a_1 * E_{\Delta ZÖPNV} + a_2 * E_{\Delta Z} + a_3 * E_{GS} + a_4 * E_{GYM} + a_5 * E_{LM} + a_6 * E_{SM} + a_0 \quad (2.1)$$

The coefficients a_1 to a_6 are the influencing factors that have to be estimated and a_0 is the residuum. A problem is the fact that the impact of the distance to the city centre on the price formation is too small in the immediate vicinity of the centre (cp. Fig. 1). There is a nearly linear connection not before the distance is bigger than about 5 km [Zeißler, 2012]. For this reason, the difference between the travel time of the particular urban district and the minimum travel time is introduced in the polynomial to achieve a proportionality between increase of distance and decrease of land values.

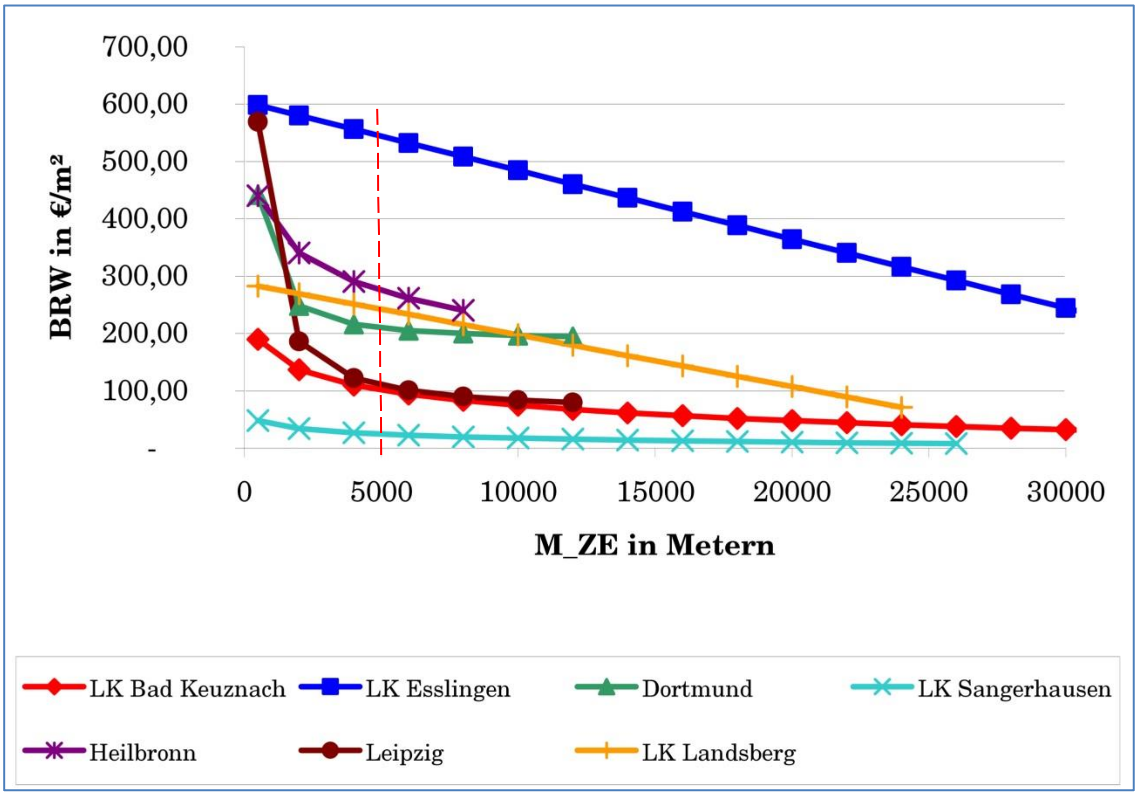


Fig. 1: Correlation between standard land value (BRW) and distance to the city centre (M_ZE) in different investigation areas (Source: Zeissler 2012, p. 183)

BW is the standard land value of the urban district in investigation, $E_{\Delta ZÖPNV}$ is the difference of distance between the urban district and the minimum distance to the city centre (measured in travel time by public transport), $E_{\Delta Z}$ is the difference of distance between the urban district and the minimum distance to the city centre (measured in travel time by car), E_{GS} is the distance to the next primary school (measured in travel time by car), E_{GYM} is the distance to the next secondary school (measured in travel time by car), E_{LM} is the distance to the next grocery (measured in travel time by car) and E_{SM} is the distance to the next supermarket that has a broader range of goods (measured in travel time by car). The impact of all other factors that are not specified explicitly in the model is included in a_0. In the following analysis has to be examined, if single parameters may be eliminated due to their small influence on the result.

The calculations are effected for spatial separated urban districts. If there are different stand land value zones in an urban district, a representative value is calculated reflecting the influence of the parameters mentioned.

Further modifications pursuant to other parameters, e.g. emissions, have to be realized by the committee of valuation experts by means of professional discretion. In doing so, the effort of data acquisition is kept on the low. This is a crucial difference to other approaches of multiple regression that try a detailed modelling of the market by using a high number of different factors. Many committees of valuation experts cannot or do not want to do the resulting work of those approaches.

The determination of the influencing factors is based on known standard land values and the corresponding distance parameters. The standard land values have to be transformed before to a homogeneous degree of land-use for building purposes (e.g. a floor space index of 0.5).

The required number of reference prices per influencing factor is equal to 15 pursuant to literature [Ziegenbein, 2010]. In praxis this amount cannot always be reached [Schmalgemeier, 1995]. However, a number as high as possible should be aspired to determine the parameters with an accuracy corresponding to the precision of standard land value determination. This precision is between +/- 20% [Schmalgemeier, 1995] and +/- 30% [Schaar, 1995] pursuant to literature.

In case of a low number of reference prices it has to be examined, if the accuracy situation may be improved by elimination of particular parameters. The regression function may be constructed therefor by using the bottom-up method. At first have to be calculated the correlations between target quantities and influencing factors and it has to be realized a first calculation of standard land values by using the highest correlated factors. In the second step has to be checked, if the result can be improved significantly by addition of further factors. The significance may be analysed by testing the regression coefficients or the coefficient of determination.

If there are sufficient reference values available, a classical regression analysis is possible by backward elimination of not significant influencing factors. On the other hand has to be taken into account that the application of statistical methods has to be combined with professional discretion of real estate valuation. This is especially important for the identification of unsuitable data [Schmalgemeier, 1995].

The design matrix $\boldsymbol{A}_{n,m}$ may be constructed by means of m=6 distances of n urban districts (denominations of matrices and vectors are printed in bold and italic to distinguish from scalars). The unknown coefficients a_i, form the vector $\boldsymbol{x}_{m,1}$ and the reference standard land values form the vector $\boldsymbol{l}_{n,1}$. The vector of residuals $\boldsymbol{v}_{n,1}$ is calculated by $\boldsymbol{v} = \boldsymbol{Ax} - \boldsymbol{l}$ pursuant to [Höpcke 1980], if all observations have the same weight.

So in $\boldsymbol{A}$ are collected the differences between distances and the corresponding mean value, in $\boldsymbol{x}$ the empiric regression coefficients, in $\boldsymbol{l}$ the differences between target quantities and mean value and in $\boldsymbol{v}$ the residuals [Hill, 2001].

The calculation of $\boldsymbol{x}$ is given by the normal equation

$$\boldsymbol{A^T A x - A^T l = 0.} \qquad (2.2)$$

With

$$\boldsymbol{A^T A = N} \text{ and } \boldsymbol{A^T l = n} \qquad (2.3)$$

follows:

$$\boldsymbol{x = N^{-1} n.} \qquad (2.4)$$

Furthermore, the following accuracy measures can be derived [Ziegenbein, 1977, pp. 31f.]:

$\boldsymbol{v^T v}$ Measure for the variation of standard land values that cannot be explained by the influencing factors and the regression model

$s_0^2 = \boldsymbol{v^T v} / (n-m-1)$ Variance (2.5)

$\boldsymbol{x^T n}$ Measure for the variation of standard land values that can be explained by the influencing factors and the regression model

$B = \boldsymbol{x^T n / l^T l}$ Multiple coefficient of determination (2.6)

The value of B is between 0 and 1. The value 1 stands for functional dependence while 0 represents the independence of target quantities from influencing factors.
But it is not enough to determine the polynomial. The basic assumptions of the multiple regression have to be verified by adequate indicators similar to B and reasonable statistical tests [Ziegenbein, 1977, p. 32]. The inner coefficient of determination B_I as well as the partial correlation coefficients between the influencing factors may be used for the evaluation of the inner structure of the approach. The partial correlation coefficients between target quantities and influencing factors provide information about the relevance of these factors.
The rows of the design matrix $\boldsymbol{A}$ may be used to calculate the confidence intervals of standard land values. If $\boldsymbol{a}_i$ is the ith row of the design matrix and $\hat{l}$ is the adjusted standard land value ($\hat{l}=l+v$), the standard deviation of the standard land values is given by [Höpcke, 1980, p. 210]

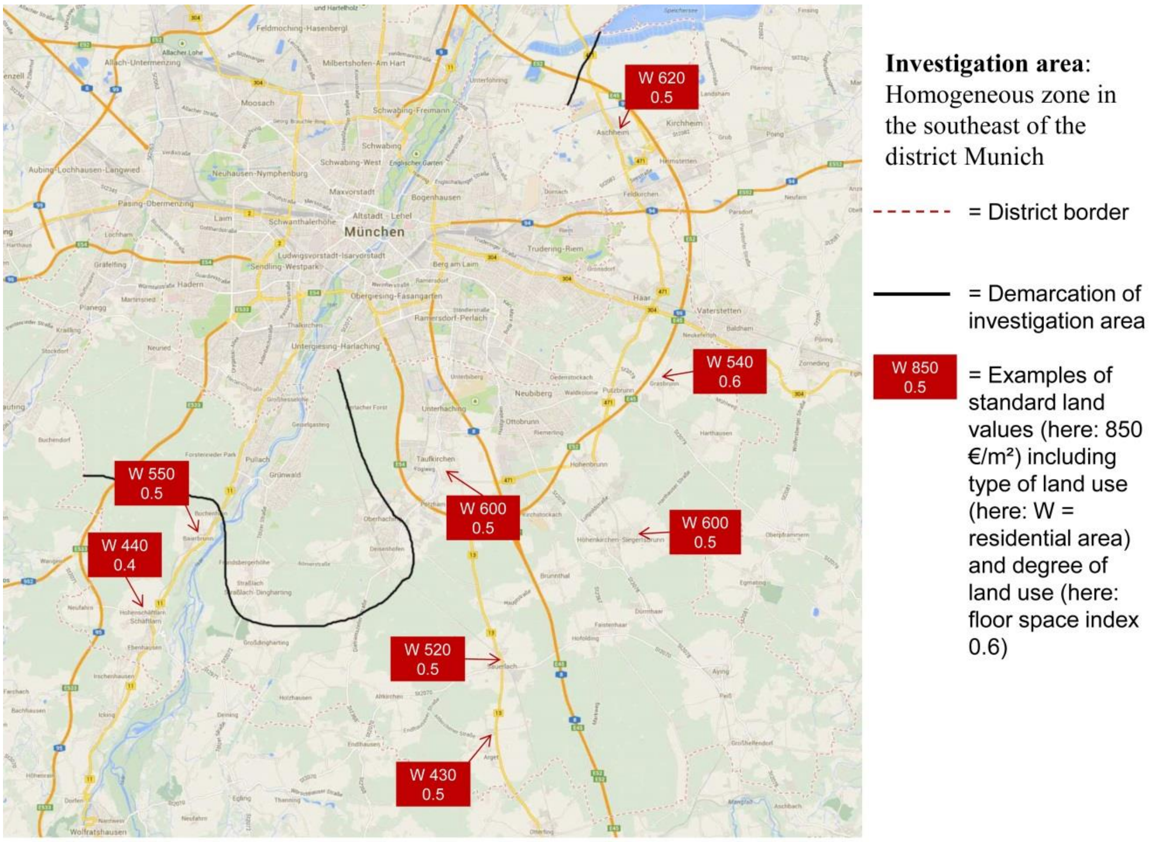
$$s_{\hat{l}_i}^2 = s_0^2 \left(1 + 1/n + \boldsymbol{a}_i \boldsymbol{N^{-1}} \boldsymbol{a}_i^T\right). \qquad (2.12)$$

The confidence interval depends on the significance level α and the degrees of freedom f of the student t distribution:

Confidence interval = $\hat{l}_i$ **+/-** $s^2_{\hat{l}_i} * t(f, 1\text{-}\alpha/2)$; $f = n - m - 1$.

3. DEMARCATION OF THE INVESTIGATION AREA

In the first step has to be checked, if an administrative district or a part of it is suitable for the application of this approach. Therefor has to be considered the map of standard land values. There must be a nearly radial structure of decreasing standard land values for increasing distances of the city centre. There may be certain differences due to the usage of travel time as parameter, but generally this structure should exist. Especially the urban districts in the boundary have to be checked critically concerning their suitability due to this simple definition of the investigation area. The tendency of decreasing standard land values for increasing distances appears clearly in the south-eastern part of the administrative district of Munich (cp. Fig. 2)



District Munich (Source: www.google.de/maps)

Fig. 2: Investigation area of standard land values in the administrative district of Munich

4. PROBLEMS OF DATA ACQUISITION

At first the travel time by car and by public transport has been registered between the particular urban districts and the city centre of Munich (Marienplatz). The travel time by car has been determined by means of the route planner of Falk (http://www.falk.de/routenplaner) as well as

Google Maps (http://maps.google.de/), which are available free of charge in the internet. The online schedule of the Munich public transport services (http://www.mvv-muenchen.de/) has been used to identify the travel time by public transport. Alternative tools are the internet presences of German Railways (www.bahn.de) and Google Maps (http://maps.google.de/).
The biggest problems concerning the definition of the travel time by public transport are different connection qualities (e.g. in comparison of working days and weekend or within and outside of rush-hours) and the modelling of non-existent access to the public transport system in particular urban districts. For this reason, travel times have been determined on Monday between 7 a.m. and 8 a.m. and between 12 a.m. and 1 p.m. and on Saturday between 2 p.m. and 3 p.m. An alternative calculation from the next connected urban district was realized, if there was no connection available in the mentioned periods. In this case 60 minutes have been added to the travel time to model the non-existing connection.
The distances to the next school, the next grocery and the next supermarket have been identified in the same way.

5. CALCULATIONS AND RESULTS

21 reference values have been available for the determination of the regression coefficients. These reference values had the highest quality ranking (reference class 1) pursuant to the committee of valuation experts because their calculation was based on at least 5 comparison prices. This low quantity of reference values is a problem because the required number of reference prices per influencing factor is equal to 15 for a reliable determination (cp. Section 2). For this reason, a complete determination of the polynomial at the beginning has just been realized to get an overview about the accuracy situation. Afterwards, the regression function has been constructed by using the bottom-up method. It has been realized a first calculation by using the highest correlated factors and in the second step had to checked, if the result could be improved significantly by addition of further factors. The standard land values have been transformed before to a homogeneous degree of land-use for building purposes (floor space index of 0.5, cp. Section 2). There have been further standard land value zones in reference class 2 (i.e. less than 5 comparison values) and in reference class 3 (without comparison values). Their values have been compared to standard land values that have been calculated by means of the polynomial. Especially the zones in class 2 are important for an additional control of the model quality.
In the first step has been realized a complete determination of the polynomial (cp. Tab. 1). The first noticeable problem is the positive algebraic sign of the coefficient of the distance to the primary school. This result pursuant to the least-squares adjustment method is heavily interpretable. According to experience, an increasing distance to the city centre, to schools or to shops should have a negative impact on the land value. The only possible interpretation of the positive sign is the fact that ths algorithm dissolves tensions within the model in this way.
In the next step has been analyzed the distance to the city centre concerning its consistency.

	Regression coefficients [€/(m²×min)]	Residuals [€/m²]	Standard land value (before) [€/m²]	Standard land value (after) [€/m²]	Urban district
a_1	-3,588	-22	620	598	Aschheim
a_2	-6,558	69	630	699	Dornach
a_3	13,952	38	510	548	Aying
a_4	-1,978	-47	550	503	Baierbrunn
a_5	-13,450	-28	549	520	Buchenhain
a_6	-2,769	11	627	638	Feldkirchen
		-4	556	553	Neukeferloh
		6	500	506	Harthausen
		1	628	629	Haar
		-39	620	581	Höhenkirchen
		-1	592	592	Luitpoldsiedlung
		58	540	598	Kirchheim
		27	590	617	Heimstetten
		-49	720	671	Oberhaching
		-3	620	617	Furth
		15	472	486	Oberbiberg
		-76	735	659	Ottobrunn (östl. Ros.)
		-2	570	568	Putzbrunn
		27	520	547	Sauerlach
		19	483	502	Ebenhausen/Zell
		1	683	685	Unterhaching
	$v^T v$ [(€/m²)²]	$x^T n$ [(€/m²)²]	B [-]	s_0^2 [(€/m²)²]	
	24916,9168	80890,6666	0,7645	1779,7798	

Tab. 1: Results in consideration of all influencing factors

The bottom-up method starts by means of the distance to the city centre because this parameter should have the biggest impact on the results pursuant to theory and first investigations of the University of Hannover [Zeißler, 2012]. First calculations including the influencing parameters $E_{\Delta ZÖPNV}$ and $E_{\Delta Z}$ resulted in big residuals for Oberhaching (Centre) and Ottobrunn ($v > 100$ €, cp. Tab. 2). The situation has been checked using the map of standard land values. The land values of both urban districts are considerably higher than in comparable districts. The positive influence of the neighboured high-price district Deisenhofen may be responsible for the high standard land value of Oberhaching. The vicinity of Munich and the high level of employment in the neighboured districts Neubiberg (University der Bundeswehr München) and Neuperlach (Siemens) are rational reasons for the high value of Ottobrunn. For these reasons, both values have been well-founded eliminated of the calculation, even if the standard land value of Ottobrunn was inside of the confidence interval based on a significance level of 5 % (tolerabel differences: 115 €/m² for Oberhaching and 120 € /m² for Ottobrunn).

Regression coefficients [€/(m²×min)]	Residuals [€/m²]	Standard land value (before) [€/m²]	Standard land value (after) [€/m²]	Urban district
-1,878	6	620	626	Aschheim
-8,711	29	630	659	Dornach
	-4	510	506	Aying
	5	550	555	Baierbrunn
	-11	549	538	Buchenhain
	32	627	660	Feldkirchen
	35	556	592	Neukeferloh
	-28	500	472	Harthausen
	-13	628	615	Haar
	-40	620	580	Höhenkirchen
	3	592	595	Luitpoldsiedlung
	73	540	613	Kirchheim
	24	590	614	Heimstetten
	-126	720	594	Oberhaching
	16	620	636	Furth
	24	472	495	Oberbiberg
	-108	735	628	Ottobrunn (östl. Ros.)
	27	570	597	Putzbrunn
	55	520	575	Sauerlach
	32	483	514	Ebenhausen/Zell
	-31	683	652	Unterhaching
$v^T v$ [(€/m²)²]	$x^T n$ [(€/m²)²]	B [-]	s_0^2 [(€/m²)²]	
45991,3062	59816,2772	0,5653	2555,0726	

Tab. 2: Results in consideration of $E_{\Delta ZÖPNV}$ *and* $E_{\Delta Z}$

Furthermore has been used the test statistic

$$\hat{t} = \frac{a_i}{s_0\sqrt{N_{ii}^{-1}}} \qquad (5.1)$$

for the examination, if the calculated regression coefficients are significantly different from zero [Höpcke, 1980, p. 209]. N_{ii}^{-1} are the elements of the diagonal matrix of the inverse of N. The test statistic has to be compared with the significance threshold of the student t distribution depending on the significance level α and the degrees of freedom $f=n-m-1$. Both regression coefficients have been proved to be different from zero (test statistics were 2.79 and 3.65 and the significance threshold was 2.13).

In the remaining 19 reference zones appeared 4 bigger residuals between 48 and 56 €/m² (absolute values, cp. Tab. 3).

Regression coefficients [€/(m²×min)]	Residuals [€/m²]	Standard land value (before) [€/m²]	Standard land value (after) [€/m²]	Urban district
-1,711	-12	620	608	Aschheim
-8,564	10	630	640	Dornach
	-9	510	501	Aying
	-2	550	548	Baierbrunn
	-16	549	532	Buchenhain
	16	627	643	Feldkirchen
	18	556	575	Neukeferloh
	-12	500	488	Harthausen
	-31	628	597	Haar
	-54	620	566	Höhenkirchen
	-11	592	582	Luitpoldsiedlung
	56	540	596	Kirchheim
	9	590	599	Heimstetten
	-1	620	619	Furth
	0	472	471	Oberbiberg
	14	570	584	Putzbrunn
	48	520	568	Sauerlach
	26	483	509	Ebenhausen/Zell
	-50	683	633	Unterhaching
$v^T v$ [(€/m²)²]	$x^T n$ [(€/m²)²]	B [-]	s_0^2 [(€/m²)²]	
14248,6500	47451,4022	0,7691	890,5406	

Tab. 3: Calculation without Oberhaching and Ottobrunn

The other residuals were smaller than 32 €/m². This is an acceptable dimension taking into account the price level in the investigation area and the fact that even prices of identical plots may differ up to +/- 10% on the real estate market [Schaar, 1995]. Another argument for the acceptability is the standard deviation s_0 of around 30 €/m². Even bigger differences are not outlier in a statistical sense because the limits of the confidence interval are between 65 and 86 €/m² based on a significance

level of 5%. However, a check is indicated, if there are factual reasons in favour of a modification or elimination of the reference value. The situation in Unterhaching is similar to Ottobrunn. For this reason, the whole area of Unterhaching, Neubiberg and Ottobrunn has been excluded of the investigation area. The remaining bigger residuals are consequence of different levels of infrastructure pursuant to the expertise of the committee of valuation experts. This statement confirms generally the importance of infrastructure for standard land values.
In the next step had to be checked, if additional parameters lead to an improvement of the results. The result of this investigation was negative. The inclusion of additional parameters did not result in numerical improvements or the results were not interpretable in a reasonable way. Accordingly, the influence of further infrastructure cannot be detected by the chosen polynomial approach. It must be taken into account by professional judgement of the committee of valuation experts. On the other hand, the reduction of effort of data acquisition is an advantage as well as the increasing number of reference values per regression coefficient.

5.1 Transformation of the points in reference class 2 and 3

Standard land value (before) [€/m²]	Standard land value (after) [€/m²]	Difference [€/m²]	Urban district	Reference class
420	480	**60**	Gudrunsiedlung	2
470	524	**54**	Otterloh	2
395	523	**128**	Waldbrunn	2
485	516	**32**	Grasbrunn	2
600	603	**3**	Ottendichl	2
625	593	**-32**	Hohenbrunn	2
430	470	**40**	Arget	2
430	469	**39**	Lochofen	2
430	473	**43**	Grafing	2
329	482	**152**	Neufahrn	2
517	443	**-73**	Hailafing	2
395	467	**72**	Großhelfendorf	3
395	512	**117**	Neukirchstockach	3
400	557	**157**	Wächterhofsiedlung	3
446	451	**5**	Grasbrunner Weg	3
470	461	**-9**	Waldsiedlung	3
556	564	**7**	Waldkolonie	3
483	445	**-38**	Großdingharting	3
411	428	**17**	Kleindingharting	3
387	456	**69**	Ebertshausen	3
387	453	**66**	Holzhausen	3
387	421	**34**	Beigarten	3
600	611	**11**	Taufkirchen	3
	Sum:	**953**		

Tab. 4: Transformation of the points in reference class 2 and 3

The residuum a_0 and the calculated regression coefficients a_1 and a_2 can be used to determine standard land values (after) in reference class 2 and 3 (cp. Tab. 4). These values can be compared with thoseones that have been fixed before by the committee of valuation experts. The results are obviously worse than in reference class 1. Apart of the four three-digit differences that would have to be checked individually it is eye-catching that most of the differences are positive. The mean value is +41 €/m². There are two possible approaches to explain this effect.

On the one hand, the quantity of low standard land values in reference class 2 and 3 is much bigger than in class 1. For this reason has to be checked, if the estimations of the committee of valuation experts in this segment of the market are generally too pessimistic. It has to be taken into account that the estimations in class 2 are based on a low number of comparison values and the standard land values in class 3 have been fixed without comparison values. According to this, the estimations are uncertain.

On the other hand, the differences may be the result of the fact that class 1 does not reflect adequately the market segment of low standard land values (cp. Fig. 3). The validity of the function (calculated by means of points in class 1) ends obviously after a travel time of about 30 minutes. The travel time in class 2 and 3 is oftentimes between 30 and 60 minutes. Due to these extrapolations the function is deemed to be unsuitable. In the next step has to be checked, if the results can be improved, if the points of reference class 2 are included in the determination of the regression coefficients. Another advantage is the increasing number of reference values per regression coefficient.

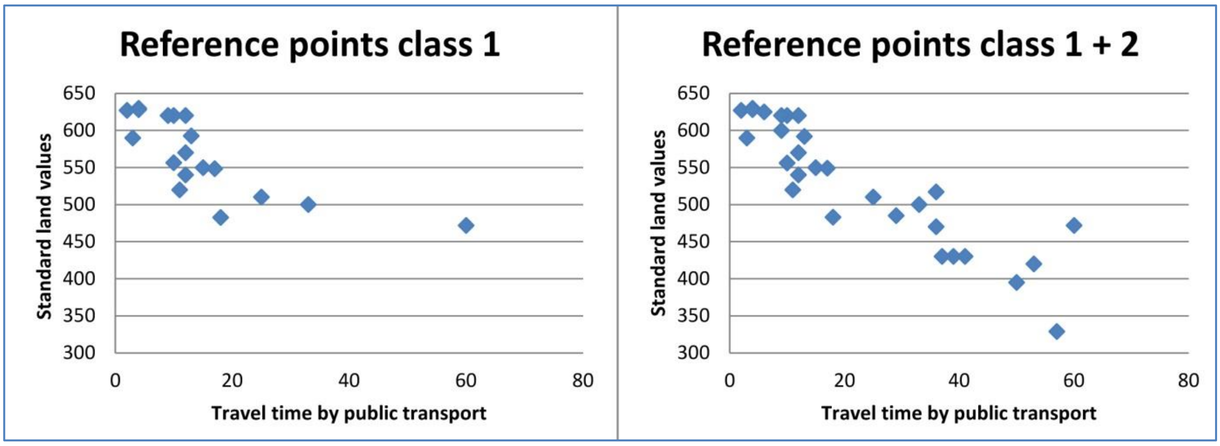


Fig. 3: Scatter diagram of standard land values and the travel time by public transport for the determination of regression coefficients by means of reference points in class 1 and in class 1 and 2

The results of the new calculation are represented in Tab. 5. An evident improvement is the elimination of very big differences. This effect is realized by a higher weighting of the travel time by public transport. While a_1 and a_2 were equal to -1,675 and -7,629 in case of the calculation by means of reference points in class 1, these quantities are equal to -3,990 and -2,084 in case of the calculation by means of class 1 and 2. For this reason, the points in class 2 should be included in the calculation of the regression coefficients to increase the influence of those urban districts that have a worse access to public transport. This result shows clearly the importance of an analysis by scatter diagrams to determine the validity interval of the polynomial.

Regression coefficients [€/(m²×min)]		Residuals [€/m²]	Standard land value (before) [€/m²]	Standard land value (after) [€/m²]	Urban district
a_0	633	-32	620	588	Aschheim
a_1	-3,990	-13	630	617	Dornach
a_2	-2,084	-2	510	508	Aying
		5	550	555	Baierbrunn
		-4	549	544	Buchenhain
		-2	627	625	Feldkirchen
		24	556	581	Neukeferloh
		-22	500	478	Harthausen
		-22	628	606	Haar
		-50	620	570	Höhenkirchen
		-22	592	571	Luitpoldsiedlung
		38	540	578	Kirchheim
		20	590	610	Heimstetten
		-26	620	594	Furth
		-96	472	376	Oberbiberg
		5	570	575	Putzbrunn
		54	520	574	Sauerlach
		52	483	535	Ebenhausen/Zell
		-13	420	407	Gudrunsiedlung
		9	470	479	Otterloh
		33	395	428	Waldbrunn
		17	485	502	Grasbrunn
		-5	600	595	Ottendichl
		-22	625	604	Hohenbrunn
		25	430	455	Arget
		30	430	460	Lochofen
		16	430	446	Grafing
		65	329	394	Neufahrn
		-61	517	456	Hailafing
	$v^T v$ [(€/m²)²]	$x^T n$ [(€/m²)²]	B [-]	s_0^2 [(€/m²)²]	
	35082,5325	8237314,25	0,9958	1403,3013	

Tab. 5: Results in consideration of class 1 and 2

Another problem is the significance test of the regression coefficients. The test statistic of a_2 is equal to -1,099. The null hypothesis ($a_2 = 0$) would only be refused for a low significance level of 28%. Generally, a significance level of 5% is applied for significance tests. Accordingly, even the travel time by public transport seems to provide such a good explication of the standard land values that cannot be improved by the travel time by car. The big influence of this parameter reflects the big differences of qualities of connections by public transport that in the investigation area exist.

The travel time can be very different depending on the connection quality by underground, railway or bus and the general existence of access to public transport. In comparison the differences of travel times by car are quite small. The elimination of a_2 requires a new calculation. The results are represented in Tab. 6.

Regression coefficients [€/(m²×min)]		Residuals [€/m²]	Standard land value (before) [€/m²]	Standard land value (after) [€/m²]	Urban district
a_0	624	-38	620	582	Aschheim
a_1	-4,212	-23	630	607	Dornach
		9	510	519	Aying
		11	550	561	Baierbrunn
		4	549	553	Buchenhain
		-11	627	616	Feldkirchen
		27	556	583	Neukeferloh
		-13	500	487	Harthausen
		-21	628	607	Haar
		-46	620	574	Höhenkirchen
		-23	592	569	Luitpoldsiedlung
		34	540	574	Kirchheim
		22	590	612	Heimstetten
		-34	620	586	Furth
		-100	472	371	Oberbiberg
		4	570	574	Putzbrunn
		58	520	578	Sauerlach
		66	483	548	Ebenhausen/Zell
		-19	420	401	Gudrunsiedlung
		3	470	473	Otterloh
		17	395	412	Waldbrunn
		17	485	502	Grasbrunn
		-15	600	585	Ottendichl
		-27	625	599	Hohenbrunn
		31	430	461	Arget
		38	430	468	Lochofen
		20	430	450	Grafing
		56	329	386	Neufahrn
		-45	517	471	Hailafing
	$v^T v$ [(€/m²)²]	$x^T n$ [(€/m²)²]	B [-]	s_0^2 [(€/m²)²]	
	36778,8896	8235617,89	0,9956	1414,5727	

Tab. 6: Results in consideration of class 1 and 2 and after elimination of a_2

The results in class 1 are similar. The multiple coefficient of determination and the variance diverge only slightly. Under these circumstances should be used only the travel time by public transport.
The maximum differences in class 2 and 3 are much smaller than before (cp. Tab. 6 and 7). The accuracy of standard land values is between +/- 20 and +/- 30% pursuant to literature. Accordingly, differences up to around 100 €/m² do not refute the model. Only the difference in the urban district Wächterhofsiedlung is too big to be explained by the used polynomial. The determined standard land value of the committee of valuation experts seems to be too small despite of the bad infrastructure. However, the "real" market value cannot be defined without comparison prices. The reduction due to infrastructure is also quite big in Großhelfendorf, but it is barely comparable to reductions in other districts.

Standard land value (before) [€/m²]	Standard land value (after) [€/m²]	Difference [€/m²]	Urban district	Reference class
395	494	**99**	Großhelfendorf	3
395	414	**19**	Neukirchstockach	3
400	595	**195**	Wächterhofsiedlung	3
446	367	**-79**	Grasbrunner Weg	3
470	371	**-98**	Waldsiedlung	3
556	582	**26**	Waldkolonie	3
483	457	**-26**	Großdingharting	3
411	451	**41**	Kleindingharting	3
387	485	**98**	Ebertshausen	3
387	477	**89**	Holzhausen	3
387	454	**67**	Beigarten	3
600	624	**24**	Taufkirchen	3
	Sum:	**454**		

Tab. 7: Transformation of the points in reference class 3 after elimination of a_2

Furthermore, considerable differences are given in the municipality of Straßlach-Dingharting including the districts Hailafing, Großdingharting, Kleindingharting, Ebertshausen, Holzhausen and Beigarten. They are consequence of different determinations of standard land values by the committee of valuation experts, although the travel time to the city centre is similar. The reason is different quality of infrastructure. The mean value of the bigger districts Großdingharting and Kleindingharting confirms the value calculated by the polynomial.
The negative values of Grasbrunner Weg and Waldsiedlung are result of the bad quality of the access to the public transport system.

6. CONCLUSION

The influence of the distance to the city centre is so big that the inclusion of additional parameters does not result in a qualitative improvement of the algorithm. In the investigation area even the

travel time by car can be eliminated due to the big differences of qualities of connections by public transport in the administrative district of Munich. This circumstance should be checked first in other investigation areas. On the other hand, the reduction of regression coefficients increases the number of reference values per parameter and in consequence the reliability of the results.
The chosen reference values should cover the whole spectrum of stand land values and travel times. 60 minutes is an adequate addition to model a non-existing access to the public transport system.
Overall the algorithm provides convincing results taking into account the accuracy of standard land value determination. The standard land values of urban districts in reference class 1 are very good modelled by the multivariate polynomial. There are bigger differences in other districts, but these discrepancies indicate that further examinations of these values are necessary. Divergences can be oftentimes explained by a very good or very bad infrastructure. The big advantage of this approach is the fact that only the impact of infrastructure has to be determined by means of professional discretion or other imprecise methods. The biggest part of price formation can be explained by the distance to the city centre with high accuracy and relative low effort. For this reason, this method can be an interesting tool for the committees of valuation experts to improve the quality of standard land values or for their first definition, if the variance of existing comparison values in a standard land value zone is too big or comparison values do not exist. Furthermore, it is an international method that can be used in any country where references values are available for the calculation of the needed coefficients of the polynomial.

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BIOGRAPHICAL NOTES

Andreas Hendricks is a scientist at the Universität der Bundeswehr München. He studied geodesy at the University of Karlsruhe where he graduated in 1994. Afterwards he worked two years in the cadastre administration and took the state examinations in 1996. After several years working for the University of Stuttgart he moved to the University of Darmstadt where he gave lectures in the areas of cadastre, real estate valuation and land management and where he finished his PhD thesis in 2006. After two and a half years at the University of Buenos Aires he moved to Neubrandenburg where he was responsible for the areas of cadastre and rural land management. Since 2012 he is postdoc at the Universität der Bundeswehr München.

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