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The Estimation of Geodetic Datum Transformation Parameters

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Research objectives

- Accuracy estimation of transformation parameters between geocentric and reference datums of Russia
- Factors determining the accuracy of transformation parameters
- The influence of nodes geometry on the accuracy of transformation parameters.

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Special conditions for Russia

- A large area (1/9 part of the inland)
- The increase of coordinate errors with distance
- Local deformations of coordinates

It is almost impossible to describe accurately the coordinate transformation by a single set of global parameters

- Large values of transformation parameters between the reference and geocentric frames: translation components are over 100 m, rotation angles are within the tenths of seconds

The linearized transformation model errors

equal to 0.1 mm.

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National Geodetic Datums in Russia

- Reference **SK-42,** based on the non-geocentric Krassovsky ellipsoid of 1940
- Reference **SK-95**, "improved" SK-42
- Geocentric **PZ-90.11** (the Earth parameters for 1990), used for ballistic and navigation tasks of GLONASS
- Geocentric GSK-**2011** introduced since 28.12.2012 for surveying and cartographic works.

The transferring of all geodetic and cartographic data into high-precision geocentric datum

is planned to finish by 2017.

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The accuracy of SK-95 and GSK-2011

SK-95	GSK-2011
Relative points position (RMS): **2 - 4 cm** for neighboring astrogeodetic network (AGN); **30 - 80 cm** for 1 - 9 K km distances.	**Corresponds to ITRF**
Elevations (RMS): **6 - 10 cm** by Class I and II leveling networks adjustment (on average in Russia).	
Exceeding Quazi-geoid heights (RMS): **6-9 cm** for 10-20 km; **30 – 50 cm** for 1 K km(astro-gravity method).	

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Transformation parameters for SK-95 – GSK-2011

Gorobets V.P., ***Definition of geocentric coordinates system and SK-95 matching.*** R&D digest Physical Geodesy. FGBU Geodesy, Cartography and GDI. 2014, pp. 95 – 98.

	Russian Federation	European Russia	Central Russia	East Russia
δX, m	25,971	25,482	23,350	22,292
δY, m	-134,831	-134,856	-130,705	-134,018
δZ, m	-80,701	-83,583	-81,080	-83,712
ω_X,"	0,139	0,071	0,008	0,073
ω_Y,"	0,038	0,030	-0,016	-0,082
ω_Z,"	-0,165	-0,229	-0,082	-0,089
$\mu \cdot 10^6$	-0,194	0,2142	-0,4277	0,1705
mB, m	±0,43	±0,16	±0,20	±0,34
mL, m	±0,37	±0,16	±0,20	±0,37

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The mathematical model of the coordinate transformation

7-parameter transformation (Helmert model):

$$\delta\mathbf{R}_2 + \breve{\mathbf{R}}_{1i}\boldsymbol{\omega} + \mu\mathbf{R}_{1i}\cdot 10^{-6} = \mathbf{R}_{2i} - \mathbf{R}_{1i}$$

Differential model:

$$1)\ \Delta\breve{\mathbf{R}}_{1i}\boldsymbol{\omega} + \mu\Delta\mathbf{R}_{1i}\cdot 10^{-6} = \Delta(\mathbf{R}_{2i} - \mathbf{R}_{1i}),$$

$\hat{\boldsymbol{\omega}}, \hat{\mu}$ – estimated values,

$$2)\ \delta\mathbf{R}_2 = \mathbf{R}_{2i} - \mathbf{R}_{1i} - (\breve{\mathbf{R}}_{1i}\hat{\boldsymbol{\omega}} + \hat{\mu}\mathbf{R}_{1i}).$$

$i = 1..n$

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The condition number of coefficient matrix

$$\mathbf{Ax} = \boldsymbol{f}$$

$$\frac{\|\mathbf{x}(\varepsilon) - \mathbf{x}\|}{\|\mathbf{x}\|} = cond(\mathbf{A})(\rho_A + \rho_f) + 0(\varepsilon^2) \quad (1)$$

ρ_A, ρ_f – relative errors of $\mathbf{A}$ and $\mathbf{f}$

$cond(\mathbf{A}) = \|\mathbf{A}\|\cdot\|\mathbf{A}^{\#}\|$ (2), $\mathbf{A}^{\#}$ - pseudo inverse matrix

$cond(\mathbf{A}) = \|\mathbf{A}\|\cdot\|\mathbf{A}^{-1}\|$ (3), $[cond(\mathbf{A})]^2 = \|\mathbf{A}^T\mathbf{A}\|\cdot\|(\mathbf{A}^T\mathbf{A})^{-1}\|$ (4).

$$k_{\delta\mathbf{R}} = \frac{\|\mathbf{m}_{\delta\mathbf{R}}\|/\|\delta\mathbf{R}\|}{(\rho_A + \rho_f)}\ (5) \qquad k_{\mu} = \frac{m_{\mu}/\mu}{(\rho_A + \rho_f)}\ (7) \qquad k_{\omega} = \frac{\|\mathbf{m}_{\omega}\|/\|\omega\|}{(\rho_A + \rho_f)}\ (6)$$

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Analysis of condition numbers and estimation of transformation parameters

Estimation criterion are:

- *Cond*(**A**), $k_{\delta R}$, k_{ω},k_{μ}
- Transformation parameters δR, ω, μ and their deviations from standard values $(\delta R - \delta R_0)$, $(\omega - \omega_0)$, $(\mu - \mu_0)$
- Weighted RMS values
- Covariance matrix and RMS errors of parameters
- Parameter variations at different perturbations in coordinates.

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Initial datasets

Territory	Diameter, km	Notes	Perturbations in the coordinates, cm
Local	35	Ordinary GNSS Network	± 4
Regional	700	Novosibirsk Region	± 25
National	6 000	Russia	± 30
Global	The whole Earth	Nodes at the Earth poles and distributed along the Equator	± 40

The simulated coordinates were perturbed by a random number generator with the values corresponding to the root-mean-square error for points position in SK-95 for the defined distances

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Cond(A) for various datasets and mathematical models

Defined parameters	Territory			
	Local (up to 35 km)	Regional (up to 700 km)	National (5-6K km)	Global
δR, ω, μ	2.61E+4	1,529E+3	2.08E+2	4.41E+1
δR, ω	2.53E+4	1.504E+3	2.06E+2	4.37E+1
ω, μ	5.53	2.50	5.94	1.74
δR	not calculated	1.73	not calculated	not calculated

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The results of transformation parameter estimation for the Helmert model

Parameter	Territory	Local		Regional		National		Global	
	Standard values	Deviation from standard values	RMS error	Deviation from standard values	RMS error	Deviation from standard values	RMS error	Deviation from standard values	RMS error
δX, m	-25	-19.930	16.893	-2.635	3.291	0.098	0.465	0.190	0.187
δY, m	131	-8.256	15.522	3.033	4.115	-0.144	0.944	-0.002	0.187
δZ, m	81	-12.626	13.876	1.761	3.506	0.243	0.635	-0.167	0.187
ω_X."	0.35	0.022	0.550	-0.046	0.153	0.009	0.069	0.007	0.007
ω_Y."	0.8	-0.212	0.503	-0.087	0.103	-0.013	0.037	0.006	0.007
ω_Z."	0.2	0.765	0.446	0.048	0.089	-0.014	0.015	0.004	0.007
$\mu \cdot 10^6$	0.1	2.58	1.88	-0.473	0.403	-0.046	0.014	0.010	0.029
RMS, m			0.077		0.334		0.481		0.488
$(\rho_A+\rho_I)$			1.8E-4		8.1E-4		1E-03		1E-03

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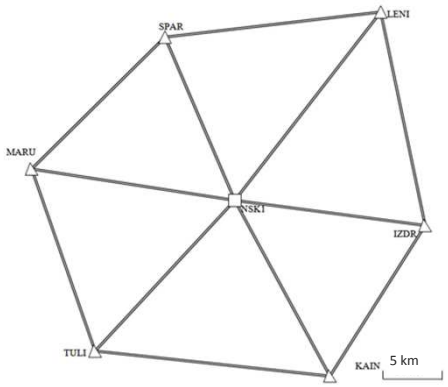
Local territory



Cond(A) = 26 100

$k_{\delta R}$= 890
k_{ω} = 5300
k_{μ}= 100 000

RMS = 0,077 m
$(\rho_A+\rho_f)$ = 1.8E-04

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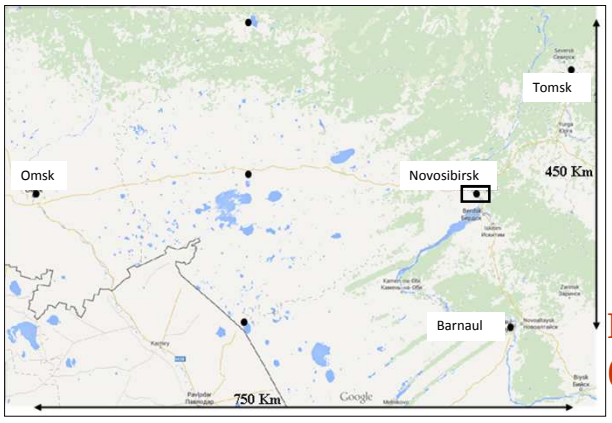
Regional territory



Cond(A)=1 529

$k_{\delta R}$= 49
k_{ω} = 270
k_{μ}= 5 000

RMS = 0,334 m
$(\rho_A+\rho_f)$ = 8.1E-04

• - Common points; ▭ - Local territory.

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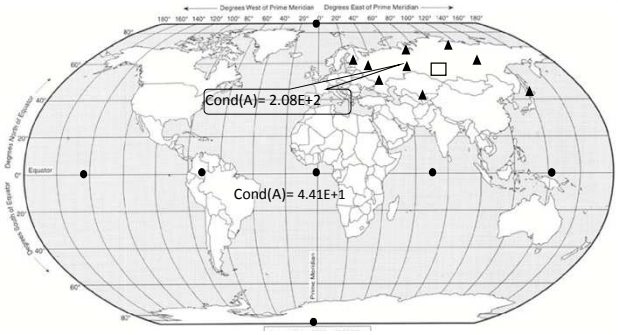
National and global territories



• – common points for **global territory**;

▲ – common points for **national territory**;

□ – **regional territory**.

National

$Cond(A) = 208$
$k_{\delta R} = 7$
$k_{\omega} = 88$
$k_{\mu} = 140$

RMS = 0,481 m
$(\rho_A+\rho_f)$ = 1E-03

Global

$Cond(A) = 44$
$k_{\delta R} = 2$
$k_{\omega} = 10$
$k_{\mu} = 290$

RMS = 0,488 m
$(\rho_A+\rho_f)$ = 1E-03

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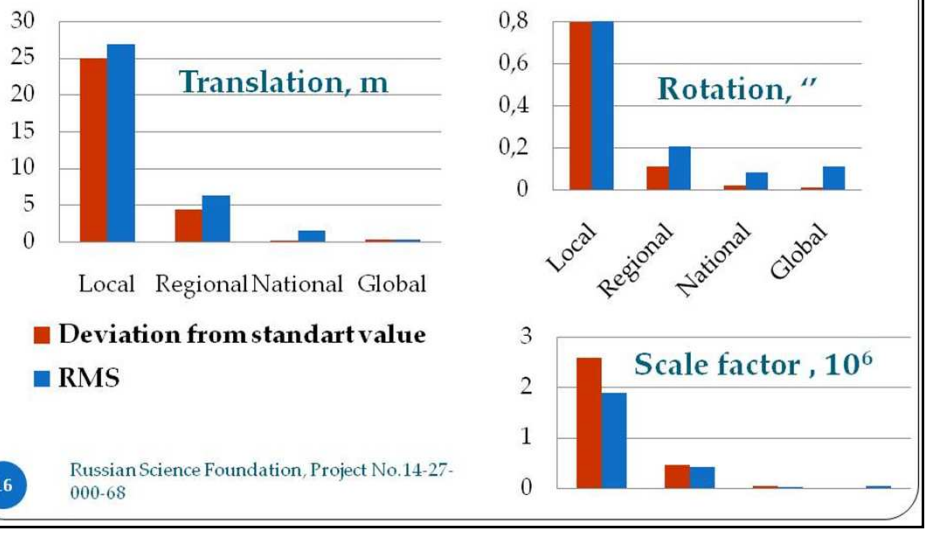
The estimation of parameters for difference territories



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Parameters variations at different perturbations in the coordinates

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Parameters	Value Differences	Total RMS	Value Differences	Total RMS
δX, m	-3.460	4.099	0.438	0.558
δY, m	1.399	5.125	0.427	1.132
δZ, m	**6.804**	**4.367**	0.241	0.761
ω_X."	0.099	0.191	-0.019	0.073
ω_Y."	-0.090	0.128	-0.009	0.038
ω_Z."	0.100	0.111	**-0.025**	**0.018**
m·10^6	**-0.976**	**0.502**	**-0.122**	**0.048**
RMS, m	0.290		0.400	

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CONCLUSION

- Deviations between estimation values of transformation parameters for different territories are mainly explained by the coefficient matrix sensitivity to the initial data errors
- The scale factor estimation is under the maximum influence
- When determination of parameters is divided into two parts (i.e. determination of **w**, m by the difference model and further estimation d**R**), we get the same results as those for the estimation by the Helmert model
- The matching parameters determined for a given territory can differ from the global parameters significantly. Nevertheless, they provide RMS error of coordinate transformation according to the initial data accuracy
- The cond(**A**) is useful for pre-estimation of nodes geometry and determination of predictable dispersion index of parameter estimations for the given territories

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