The Estimation of Geodetic Datum Transformation Parameters

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SUMMARY

The accuracy estimation of transformation parameters between geocentric and reference datum that is the national geocentric reference system 2011(GRS-2011) of the Russian Federation and CS-95 coordinate system (CS) has been done. Describes the factors that determine the accuracy of the transformation parameters: the accuracy of the input datasets, missing precise heights in a reference system, geometry of common points location and territory size.

The last factor is described by the condition number of the system of equations cond(A), and does not depend on the errors of the input data. In this work the study of condition number variation and coordinate transformation parameters estimation errors variation with the given mathematical model and with the dataset area has been performed. For experiments was simulated multiple point sets in both coordinate systems. The point sets occupied by several different sizes of areas: from the local, the size of an ordinary satellite network (35 km in diameter), to global, covering the whole Earth. The basic mathematical transformation model - static Helmert model with 7 parameters, that were close to the published transformation parameters; then the given parameters were considered as standard ones. In the model coordinate values are made perturbations at the level of the real errors of coordinate points.

In article presents the results of the determination cond(A) and transformation parameters for several mathematical models and for different point sets. Criteria for analysis - is cond(A), the difference between the parameter estimates with their standard values, and Root Mean Square (RMS) obtained with residuals at the common points. Also considered factors loss of precision for individual parameter groups: translate, rotate and scale factor. It is shown that the most sensitive to errors in the input data has a scale factor, least – translate vector .

The condition number as a measure of lowering the parameter precision decreases with the increase of the layout area common points, but does not become equal to unity even for global coverage. For the territory of Russia cond(A)≈200, for the whole Earth cond(A)≈40.

Perturbations in the coordinates at the level of the RMS for the same data set lead to significant changes in the parameter estimates. The difference between the parameter estimates are within the confidence interval, asked their errors. On the local area difference can be significant (up to 10 meters for a translate of the coordinate origin), but to ensure the transformation of the point coordinates within the field of approximation with accuracy corresponding to the measurement RMS (1 - 10 cm). These parameters, called matching, are appropriate for this area.

The parameters differ from the standard values, obtained on simulated data for regional areas are of the same order as the differences between the actual estimates of the parameters obtained for different regions of Russia.

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1. INTRODUCTION

The implementation of the geocentric coordinate system GRS-2011 [7] on the territory of Russia makes it important to define the coupling data between this coordinate frame that is using satellite positioning to the advantage and the national reference CS-95 [8]. Here the main task is to use Global Navigation Satellite System(GNSS) data advantages in full together with the existing geodetic and cartographic media published in the reference coordinate system. Coordinate transformation from the geocentric system to the reference one must be performed without loss of high GNSS data accuracy.

In theory direct coordinates transformation from a geocentric system to the reference one with precise coupling parameters according to the Helmert model (similarity transformation) would provide a strict and accurate coupling between the coordinate systems, help to have precise positioning in the reference system in Real Time Kinematic (RTK) mode, and resolve other contradictions of the national coordinate space (e.g. nautical charts' inconsistency).

Though in Russia in practical geodesy at GNSS data processing direct coordinates transformation from the geocentric system to the reference one with published global transformation parameters tend to be performed only in rough computations, e.g. for the following constrained adjustment of GNSS Network holding some of its points in the reference system. In databases of various GNSS data processing software different transformation parameters are used, and they sometimes do not correspond to the published ones. At present Continuously Operating Reference Stations (CORS) that are the national geocentric coordinate system do not cover all the vast territory of Russia. So to perform constrained adjustment of the GNSS Network additional measurements are required at a number of national geodetic network points that are RS coordinate carriers with data taken in hostile environment for GNSS measurements. Besides deformation of GNSS Network at the constrained adjustment, uncertainties of transformation parameters imply adjustment results' deviations. A lot of surveyors estimate local coordinate transformation parameters for limited areas [10], [13], [1].

Coordinate transformation accuracy depends on transformation parameters precision and mathematical model correctness. Whereas transformation parameters' estimation precision comes under the influence of the following factors:

1. Source data precision (i.e.GNSS reference stations' relative position precision) contrast is by (at least) an order of magnitude greater than corresponding precision of the current national geodetic network of Russia. CS-95 [2] precision is characterized by relative points position' RMS of 2 - 4 cm for neighboring astrogeodetic network (AGN) and 0.3 - 0.8 m for 1 - 9 K km distances. Elevations' precision depends on the measuring method and is characterized by RMS of 6 - 10 cm by Class I and II leveling networks adjustment (on average in Russia), and of 0.2 - 0.3 m – by astrofixes at AGN creation. Quazi-geoid heights'

gain precision at astro-gravity method is characterized by RMS of 6-9 cm for 10-20 km distances and 0.3 - 0.5 m for 1 K km.

2. Missing precise heights in a reference system after separation of the national coordinate frame that has been created by surface techniques into plans and elevations. Datums that are carried out by satellite techniques form a 3D spatial construction with roughly similar coordinate precision. This factor has been noted in a lot of published works [8], [12], [13], [15]. As it is obvious from [9], [5], parameter estimation errors connected with heights uncertainties influence mainly the scale parameter value;

3. The common points geometry with known coordinates that specifies coefficient matrix sensitivity to initial errors. This factor defines coupling parameters estimation precision for any coordinate frames (including both geocentric and reference ones). A limited area leads to an ill-conditioned coefficient matrix of the mathematical model.

The aim of the present work is to estimate the potential precision assessment of coupling parameters for geocentric and Earth coordinate systems, to specify factors that influence the precision, and to recommend on the transformation parameters definition and use.

2. THE INFLUENCE OF THE GEOMETRY OF THE COMMON POINTS LOCATION ON THE TRANSFORMATION PARAMETERS PRECISION ESTIMATION

2.1 The mathematical model of the coordinatetransformation

The source data for transformation parameters definition tend to be the coordinates of common points in two coordinate systems, as well as the difference of point pairs coordinates (baseline components) from GNSS data.

Let's assume that $\mathbf{R}_1 = [X_1 \quad Y_1 \quad Z_1]^T$ – a radius vector in the first CS, and $\mathbf{R}_2 = [X_2 \quad Y_2 \quad Z_2]^T$ – a radius vector in the second CS.

The equations system to define transformation parameters using the Helmert model [6] for *n* common points is as follows:

$$\boldsymbol{\delta}\mathbf{R}_2 + \breve{\mathbf{R}}_{1i}\boldsymbol{\omega} + \mu\mathbf{R}_{1i} \cdot 10^{-6} = \mathbf{R}_{2i} - \mathbf{R}_{1i},\ i = 1 \ldots n, \quad (1)$$

where $\delta\mathbf{R}_2 = [\delta X_2 \quad \delta Y_2 \quad \delta Z_2]^T$ is vector of translation (origin O_1displacement of the first CS with respect to the origin O_2 of the second CS),

$\omega = [\omega_x \quad \omega_y \quad \omega_z]^T$ – rotation vector of the second CS with respect to the first one, in arc seconds,

μ – scalefactor, in units multiplied by 10^6,

$\breve{\mathbf{R}}_1 = \frac{1}{\rho''}\begin{bmatrix} 0 & -Z_1 & Y_1 \\ Z_1 & 0 & -X_1 \\ -Y_1 & X_1 & 0 \end{bmatrix}$ - coefficient matrix at the defined CS rotation ω,

where ρ'' is the number of arc seconds in one radian.

In case we take the difference of point pairs coordinates as the source data for transformation parameters definition, the vector $\delta\mathbf{R}$ will be eliminated from equations [14]:

$$\breve{\Delta\mathbf{R}}_{1i}\boldsymbol{\omega} + \mu\Delta\mathbf{R}_{1i} \cdot 10^{-6} = \Delta(\mathbf{R}_{2i} - \mathbf{R}_{1i}),\ i = 1\ldots n, \qquad (2)$$

whereΔ denotes the coordinates difference for two points in the first and second system.

The determined estimation of rotation parameters $\hat{\boldsymbol{\omega}}$ and scale factor $\hat{\mu}$ that are transferred into the right part of the Helmert model the translation vector components are defined with the help of equations set of the type

$$\boldsymbol{\delta}\mathbf{R}_2 = \mathbf{R}_{2i} - \mathbf{R}_{1i} - (\breve{\mathbf{R}}_{1i}\hat{\boldsymbol{\omega}} + \hat{\mu}\mathbf{R}_{1i}),\ i = 1\ldots n. \qquad (3)$$

CS transformation parameters definition is based on the solution of linear equations of the type (1), (2), or (3), with covariance matrix computation to estimate parameters precision. Covariance matrix diagonal includes RMS errors' squares for parameters estimation.

Now for the relationship between the dynamic coordinate systems (for example, different implementations of the ITRF) is used 14-parameter transformation, where an additional 7 parameters - the first time derivatives, [3], [11], [17].As the coordinate system GSK-2011 is static, (see Russian Standard "Coordinate System", [6]) the accuracy estimation of only 7 transformation parameters, without their first times derivations, has been don.

2.2 The condition number of coefficient matrix

The mathematical models of the coordinate transformation (1), (2), (3) can be written as system of linear equation **Ax=f**, where **x** is the vector of unknown parameters, **A** - coefficient matrix, **f** –vector of right part.
Errors for unknown **x** in the equations system **Ax=f** generally depend on the system sensitivity to initial errors; errors for transformation parameters depend on the geometry of common points location and the distance between the nodes.

The sensitivity of the equations linear system **Ax=f** to the coefficient matrix perturbations is specified by the condition number [4]; for square matrixes is calculated by the formula

$$\mathrm{cond}(\mathbf{A}) = \|\mathbf{A}\|\,\|\mathbf{A}^{-1}\|\ ,$$

while for rectangular matrixes, in general terms,

$$\mathrm{cond}(\mathbf{A}) = \|\mathbf{A}\|\,\|\mathbf{A}^{\#}\|\ ,$$

where$\mathbf{A}^{\#}$ is a pseudo inverse of matrix.

The condition number specifies how many times the parameters estimation vector relative error **x** is larger than initial error in coefficient matrix **A** and vector **f**. At perturbations ε, [4],

$$\frac{\| \mathbf{x}(\varepsilon) - \mathbf{x} \|}{\| \mathbf{x} \|} = cond(\mathbf{A})(\rho_A + \rho_f) + 0(\varepsilon^2) \quad , \tag{4}$$

where ρ_A, ρ_f are relative errors**A** and **f**.

In the solution of the equation set there are several ways to perform a matrix inversion, i.e. QR- decomposition or SVD (singular value decomposition) [4]. On a settled basis in geodesy there is a technique of standard equations system compilation that results in the increase of a condition number. For standard equations system the condition number is found by the expression [16]

$$\text{cond}(\mathbf{A})^2 = \|\mathbf{A}^\mathbf{T}\mathbf{A}\| \, \|(\mathbf{A}^\mathbf{T}\mathbf{A})^{-1}\|.$$

In this work the study of condition number variation and coordinate transformation parameters estimation errors variation with the given mathematical model and with the dataset area has been performed.

2.3 Input datasets and mathematical models

For the purpose of the study several sets of points-coordinate carriers in two systems were simulated. Coordinate transformation from one system to another was performed according to the Helmert model with parameters that were close to the published transformation parameters; then the given parameters were considered as standard ones. Then the simulated coordinates were perturbed by a random number generator with the values corresponding to the root-mean-square error for points position in CS-95 for the defined distances [2] (see the introduction to this article).

Then the inverse problem was being solved, i.e. parameters with the errors were estimated and cond(**A**) (independent on error **f**) was calculated. In order to estimate only condition number' influence on the parameters estimation results, the performed simulation implied that the coordinate reference system had precise heights, and normal distribution of introduced measurement errors had been chosen. So the point sets were simulated for the following territories:

- local, inter-station distance of 15 - 20 km (diameter of 35 km), the area of the ordinary GNSS Network (Fig.1), perturbations in the coordinates within ± 4 cm;
- regional, the territory of the Novosibirsk Region (inter-station distance of up to 700 km) (Fig.2), perturbations in the coordinates within ± 25 cm;
- national, the territory of Russia (5-6K km) (Fig.3), perturbations in the coordinates within ± 30 cm;
- global, cover the Earth, with nodes at the Earth poles, and normally distributed along the Equator (Fig.3), perturbations in the coordinates within ± 40 cm.

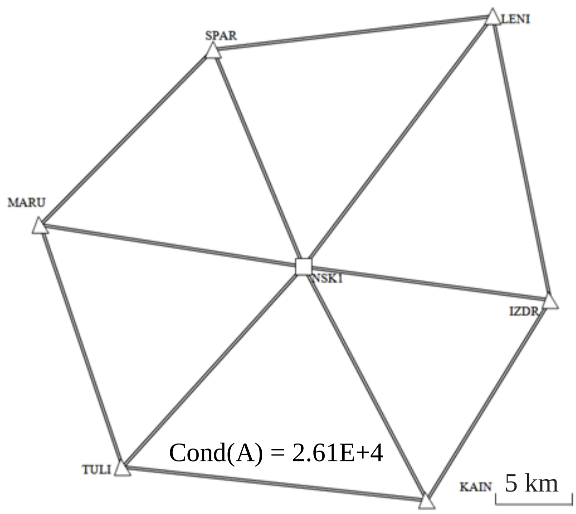

Figure 1: Local territory. GNSS Network.

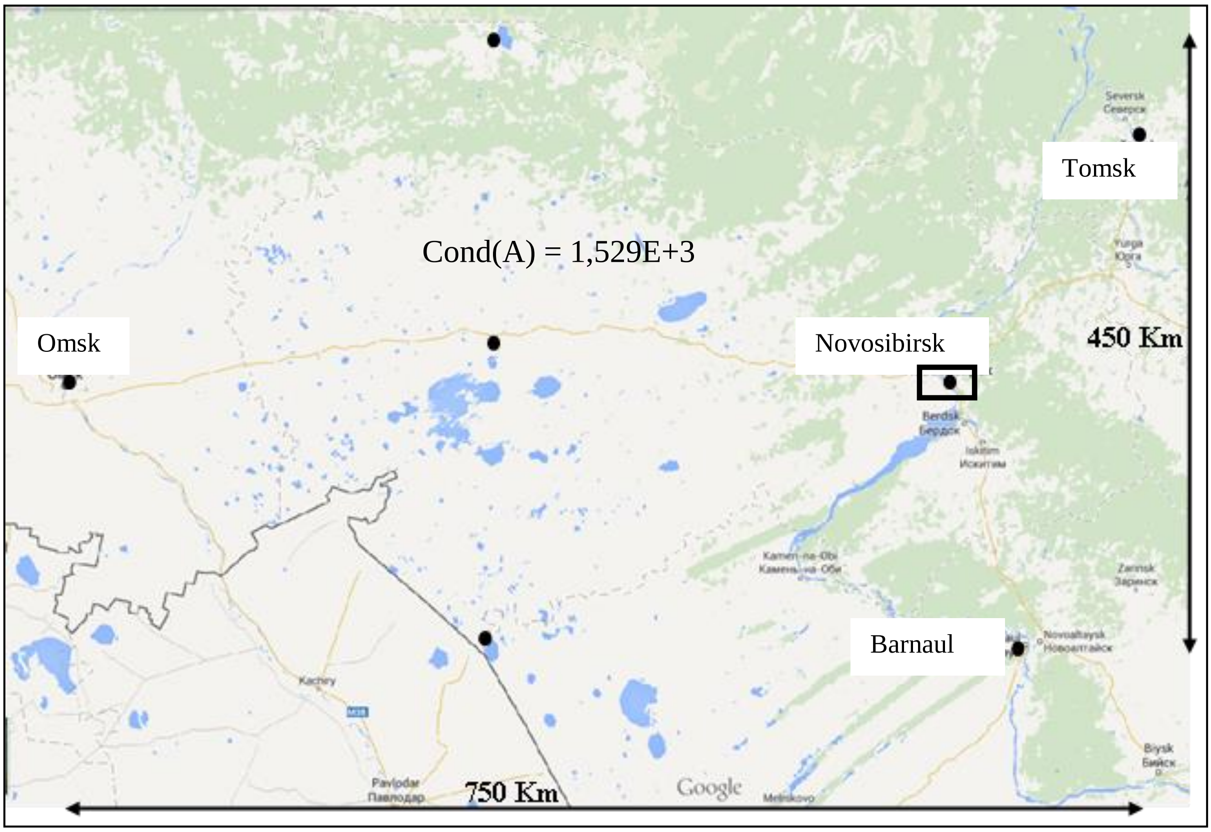

Figure 2: Regional territory. ● - common points; ▭ - Local territory.

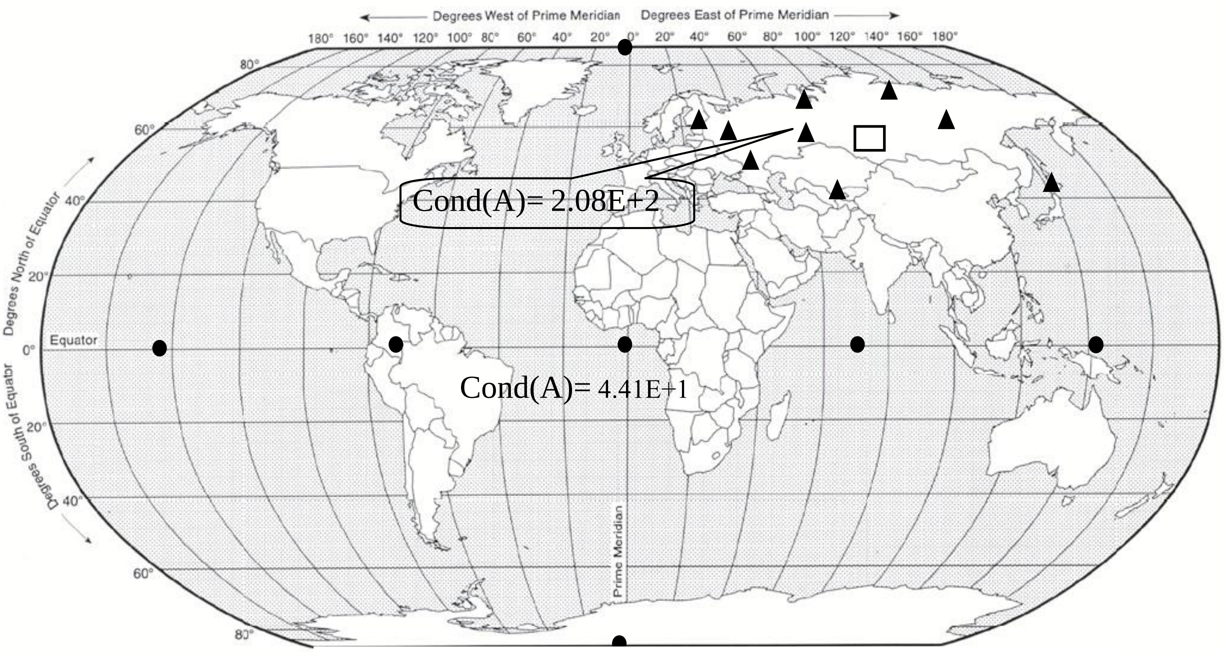

Figure 3: Global and National territory.

● – common points for Global territory;

▲ – common points for National territory;

□ – Regional territory.

Transformation parameters estimation and calculation of cond(**A**) were performed for several mathematical models:

–the Helmert model (1), estimation of seven parameters δ**R, ω**, μ;

–the Helmert model (1), estimation of six parametersδ**R, ω**, without the scale factor;

–model (2), for the difference of point pairs coordinates, excluding the translation vector from the estimation; estimation of the rotation vector and the scale factor **ω**, μ;

–model (3) with translate vector δ**R** estimation.

2.4 Results

2.4.1 Condition number

Condition number calculation results for various datasets and mathematical models are given in Table 1. Condition numbers for the 7-parameter model (1) are shown in Figures 1-3.

Table 1.Values cond(**A**) for various datasets and mathematical models

Mathematical model, defined parameters	Territory			
	Local (up to 35 km)	Regional (up to 700 km)	National (5-6K km)	Global
(1), δ**R,** ω, μ	2.61E+4	1,529E+3	2.08E+2	4.41E+1
(1), δ**R, ω**	2.53E+4	1.504E+3	2.06E+2	4.37E+1
(2), ω, μ	5.53	2.50	5.94	1.74
(3), δ**R**	Not calculated	1.73	Not calculated	Not calculated

In Table 1 it is obvious that the condition number is decreasing at the increase of the territory where transformation parameters are being defined. The scale factor has an insignificant influence on the coefficient matrix sensitivity. The coordinate axes rotation estimation ω according to model (2) is the most stable one, though for local territories the right part of the model formed by the coordinate difference becomes too small, i.e. at the level of initial errors and insignificant. For the regional territory **ω**, μ according to model (2) and δ**R** according to model (3) are defined by cond(**A**) sustainably.

2.4.2 Parameters estimation

Parameters estimation deviations from their standard values and having the deviations within the confidence interval specified by parameters' RMS errors are important indicators of the equations set solution quality. In Tables 2, 3, and 4 there are parameters estimation results according to models (1), (2), and (3) for various territories: parameters estimation deviations from their standard values, weighted root-mean-squarevalue(RMS value)calculated by residuals in common points, as well asRMS errors for parameters based on the covariance matrix. The deviations from standard values that are over RMS error are marked in bold. Here there is a right part and coefficient matrix relative error for every case ($\rho_A + \rho_f$) and parameters estimation accuracy degradation,

$$k_{\delta\mathbf{R}} = \frac{\|\boldsymbol{m}_{\delta\mathbf{R}}\| / \|\delta\mathbf{R}\|}{(\rho_A + \rho_f)}, \quad k_{\omega} = \frac{\|\boldsymbol{m}_{\omega}\| / \|\omega\|}{(\rho_A + \rho_f)}, \quad k_{\mu} = \frac{m_{\mu} / \mu}{(\rho_A + \rho_f)}.$$

The coefficients show the influence of the geometry of common points location on the estimation of certain groups of parameters. The coefficients significance is similar to the significance of the condition number for the general equations system if we consider parameters groups $\delta\mathbf{R}$**,** ω, μ independent on each other.

Table 2. Transformation parameters estimation results according to the Helmert model (1)

Parameter	Territory	Local (up to 35 km)		Regional (up to 700 km)		National (5-6K km)		Global	
	Standard values	Deviation from standard values	RMS error	Deviationfrom standard values	RMS error	Deviation from standard values	RMS error	Deviation from standard values	RMS error
δX, m	-25	**-19.930**	16.893	-2.635	3.291	0.098	0.465	**0.190**	0.187
δY, m	131	-8.256	15.522	3.033	4.115	-0.144	0.944	-0.002	0.187
δZ, m	81	-12.626	13.876	1.761	3.506	0.243	0.635	-0.167	0.187
ω_X."	0.35	0.022	0.550	-0.046	0.153	0.009	0.069	0.007	0.007
ω_Y."	0.8	-0.212	0.503	-0.087	0.103	-0.013	0.037	0.006	0.007
ω_Z."	0.2	**0.765**	0.446	0.048	0.089	-0.014	0.015	0.004	0.007
$\mu \cdot 10^6$	0.1	**2.58**	1.88	**-0.473**	0.403	**-0.046**	0.014	0.010	0.029
Weighted RMS value, m			0.077		0.334		0.481		0.488
$(\rho_A+\rho_f)$			1.8E-4		8.1E-4		1E-03		1E-03
$k_{\delta\mathbf{R}}$			8.9E+2		49		7		2
k_ω			5.3E+3		2.7E+2		88		10
k_μ			10^5		5.0E+3		1.4E+2		2.9E+2

According to the results in Table 2 it is obvious that parameters estimation deviations from the standard values decrease at the increase of the solution territory. For local territories the order of magnitude for parameters deviations is similar to the values themselves. Nevertheless the parameters that are found after the solution of the equation set and are far from the standard values provide for an acceptable weighted *RMS value* for coordinates calculation within the approximation area. Such parameters can be named “matching” ones; they are specifically used in geodesy practice to recalculate coordinates for the local territory.

All deviations of parameters estimation from standard values are within the confidence range that is specified by corresponding RMS errors (less than 2RMS); mostly the deviations are within an RMS error limit. Parameters estimation deviations from the standard values that are over RMS error appear mainly for the scale parameter.

The values of the table coefficients $k_{\delta\mathbf{R}}$, k_ω and k_μ, show that initial errors have the least significant influence on the translate vector parameter estimation, and the most significant influence on the scale factor estimation.

Table 3. Transformation parameters estimation results according to the difference model (2)

Parameter	Territory	Local (up to 35 km)		Regional (up to 700 km)		National (5-6K km)		Global	
	Standard values	Deviation from standard values	RMS error	Deviation from standard values	RMS error	Deviation from standard values	RMS error	Deviation from standard values	RMS error
ω_X."	0.35	0.088	1.122	-0.053	0.132	0.012	0.026	0.005	0.003
ω_Y."	0.8	-0.004	0.727	-0.082	0.085	**-0.011**	0.008	0.005	0.003
ω_Z."	0.2	0.546	0.691	0.087	0.071	-0.008	0.008	0.003	0.003
$\mu \cdot 10^6$	0.1	0.547	2.859	**-0.875**	0.320	-0.014	0.038	0.007	0.013
Weighted RMS value, m			0.153		0.458		0.421		0.371
$(\rho_A+\rho_f)$			0.674		0.075		8.7E-03		4.12E-03
k_ω			2.41		2.55		3.6		1.4
k_μ			2.34		42.7		43.7		51.55

Similar to the previous case, in Table 3 all deviations of parameters estimation from standard values are within the confidence range that is specified by corresponding RMS errors (less than 2RMS); mostly the deviations are within an RMS limit, excluding scale factor estimation for the regional territory.

Table 3 shows that weighted RMS value for model (2) for all the territories is of the same order of magnitude as Table 2 data (model 1), excluding local territory where the weighted RMS error is twice as much as the value specified for model (1). It is explained by low informative value of the right part of the model (2) for the limited territory. However,vector of deviations $\boldsymbol{\omega}$ from the standard values, $\|\Delta\boldsymbol{\omega}\|$, at the local territory are a bit smaller here than for model (1), i.e. 0.553" against 0.794". It is explained by the significant (three orders of magnitude) reduction of $k_{\boldsymbol{\omega}}$ coefficient**.** For the rest of the cases the difference of standard values from the calculated parameters $\boldsymbol{\omega}$ is of the same order as for model (1) with seven parameters.

Estimation of the scale factor according to model (2) is closer to the standard value if compared to model (1) in all the cases excluding estimation for the regional territory.

Coefficients k_ω for all the considered territories are of the same order with condition numbers for model (2), specified in Table 1, and for the scale factor k_μ outreach the estimated values cond(A) by an order of magnitude. Here the conclusions for model (1) are proved, i.e. the scale factor estimation is the most vulnerable to initial errors influence.

In Table 4 there are translation vector estimation results for the regional territory by model (3).

Table 4. Estimation results $\delta\mathbf{R}$ according to model (3) with previously calculated$\boldsymbol{\omega}$, μ according to the difference model (2)

	Territory	Regional	
Parameter	Standard values	Deviationfrom standard values	RMS error
δX, m	-25	-3.232	0.369
δY, m	131	4.417	0.369
δZ, m	81	3.669	0.369
Weighted RMS value, m			0.953

As it is obvious from the table translate vector deviations from the standard values is more or less of the same order of magnitude as in table 2, i.e. pre-definition of the rotation vector and the scale factor (though with smaller errors) did not improve the result. Besides weighed RMS error increased it more than twice as a result of errors based on estimation errors $\boldsymbol{\omega}$ and μ, transferred into the right part. It may be concluded that parameters estimation splitting into two steps (model 2 and 3) does not improve the result in spite of lower condition number for equations sets.

2.4.3 The sensitivity of parameter estimates to perturbations in the input data

The cond(**A**) utility is in the possibility to pre-estimate geometry of common points location and to define a predictable transformation parameters estimation span for the set territory, according to formula 4.

Table 5 and **6** data show the parameters estimation variations for regional and national territories and their dependence on the initial errors. Parameters estimation was performed for the same datasets, though with different perturbations of the same order at the coordinates with the help of a random number generator.

Table 5. Parameters estimation variations at different perturbations at the coordinates. Regional territory

	Regional Option 1		Regional Option 2		Differences of values Option 1 - Option 2	Total RMS error value $\sqrt{RMS_1^2 + RMS_2^2}$
Parameter	Value	RMS error	Value	RMS error		
δX, m	-27.635	3.291	-24.175	2.443	-3.460	4.099
δY, m	134.033	4.115	132.634	3.055	1.399	5.125
δZ, m	82.761	3.506	75.957	2.603	**6.804**	4.367
ω_X,"	0.053	0.153	-0.046	0.114	0.099	0.191
ω_Y,"	0.263	0.103	0.353	0.076	-0.090	0.128
ω_Z,"	0.847	0.089	0.747	0.066	0.100	0.111
$m \cdot 10^6$	-0.273	0.403	0.703	0.299	**-0.976**	0.502
Weighted RMS value, m		0.334		0.245		

Table 6. Parameters estimation variations at different perturbations at the coordinates. National territory

	National Option 1		National Option 2		Differences of values Option 1 - Option 2	Total RMS error value $\sqrt{RMS_1^2 + RMS_2^2}$
Parameter	Value	RMS error	Value	RMS error		
δX, m	-24.902	0.465	-25.340	0.308	0.438	0.558
δY, m	130.855	0.944	130.428	0.624	0.427	1.132
δZ, m	81.243	0.635	81.002	0.420	0.241	0.761
ω_X."	0.109	0.069	0.128	0.025	-0.019	0.073
ω_Y."	0.336	0.037	0.345	0.010	-0.009	0.038
ω_Z."	0.785	0.015	0.810	0.010	-0.025	0.018
$m \cdot 10^6$	0.153	0.014	0.275	0.046	**-0.122**	0.048
Weighted RMS value, m		0.481		0.318		

As it is obvious from Tables 5 and 6 at the weighed RMS error of the same order of magnitude small perturbations at the coordinates of the same (!) datasets result in significant parameters estimation changes. For the regional territory of up to 700 km translate vector variations are within 3 - 6 m, and for the inter-point distances of 3 - 6 K km the values are at the level of weighed RMS error. Scale factor estimations are of maximum variations that are much larger that its RMS error.

Parameters estimation changes connected with initial perturbations for the regional territory is of the same order of magnitude as differences from published in [8], at the estimation of coupling parameters between GRS-2011 and CS-95 for different parts of the Russia.

3. CONCLUSION

According to the result of the performed study it may be concluded that deviations between estimation values of global transformation parameters at different territories are explained mainly by the coefficient matrix sensitivity to the initial errors. For the local territory, parameters estimation can differ from the standard values significantly, and increase of the territory size results in minimizing of initial errors influence on the results. The scale factor estimation is under the maximum influence. Parameters estimation splitting into two steps (i.e. definition $\boldsymbol{\omega}$, μ according to the difference model (2) and estimation $\delta\mathbf{R}$ according to model (3)) has more or less the same results as the complete seven parameters' estimation by Helmert.

For strict adjustment of the geocentric and reference coordinate systems the required quasi-geoid height in relation to a reference ellipsoid (reference height anomaly) cannot be defined by a global quasi-geoid model. The transfer from global height anomaly to the reference one is performed with global transformation parameters at an error of 5 - 10 m. Such error implies the scale change of $(0.6 \div 1.2)\cdot 10^{-6}$. The deviations of the scale factor estimation from the standard value in Table 2 are of the same order of magnitude and are the results of an ill-conditioned equation set. So it may be concluded that transformation parameters estimation precision at similar initial errors is influenced mainly by the geometry of common points location, size of the territory, while a height error above the reference ellipsoid is of smaller influence.

For the defined territory one can specify different (matching) sets of transformation parameters that differ from the global ones within their errors, and that provide for coordinate transformation RMS error according to the accuracy of the initial data.

As a conclusion it should be noted that published geodetic datum transformation parameters estimations must be followed by their precision characteristics.

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