Velocity Field across the Carmel Fault Calculated by Extended Free Network Adjustment Constraints

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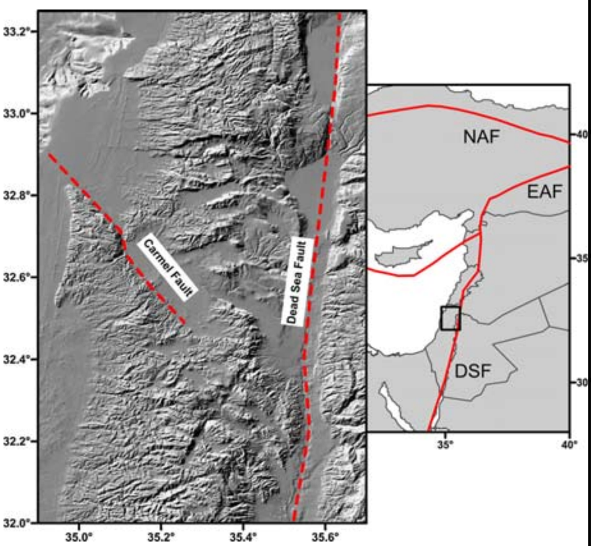
Geological Background

► The Carmel Fault is one of the major geological structures of northern Israel.

► It is part of Carmel-Tirtza Fault System.

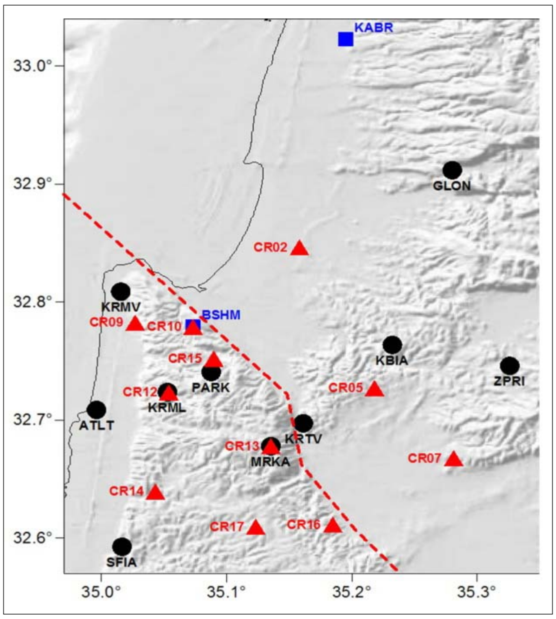
► It is branch of the Dead Sea Fault.



GPS Network

The Carmel Fault region is covered by monitoring geodetic network consists of 23 sites which were measured four times in **1999**, **2006**, **2009** and **2010** by means of GPS.

- ▲ Carmel Network
- ● G1 Network
- ■ APN Network



Field Work

- ► All sites were occupied at least twice in each campaign.
- ► The observation time spans was 4 h for the 1999 campaign and 8 h for all the other campaigns (2006, 2009 and 2010).
- ► The Carmel network points were occupied using tripods.
- ► G1 network sites were measured using special centering devices, which allow an easier setup of the antenna.

GPS Data Analysis

- ► Data from seven APN (Israeli network of permanent GNSS stations) sites and three IGS sites (NICO, ANKR and ZECK) are included in the GPS analysis.
- ► The GPS data analysis was done using the Bernese GPS Software Version 5.0.
- ► Precise ephemeris and EOPs from IGS were used.
- ► Absolute phase centre variations with respect to IGS05 standard were used for all campaigns.
- ► The datum was defined by loose constraints of the IGS and APN site coordinates.
- ► The formal variance-covariance matrices were multiplied by a scale factor of 25.

The Datum Problem

- ► In monitoring geodetic networks the deformation parameters can be estimable only if the datum of the network has not been changed between measurement epochs.
- ► Geodetic measurements can define the inner geometry of the points in the network, but they are incapable of completely determining its datum.
- ► GPS vectors in a network can define its datum parameter of orientation and scale.
- ► The remaining datum parameters are defined by imposing linear constraints on the estimated coordinate corrections.

The Datum Problem

- ► In GPS network the part of the datum definition determined by the vectors may not remain consistent.
- ► We cannot ensure that the part of the datum definition determined by the vectors is the same in each monitoring campaign and remains stable over time.

Motivation

- ► To prevent the effect of datum components which present in GPS vectors on the adjustment of a 4D network.

If an appropriate steps are not taken the result will be an inevitable mixture between the *deformation parameters* and the *datum components of the measurements*.

Proposed Solution

► To sterilize the geodetic measurements from their datum definition content.

The datum in each monitoring campaign is defined in its entirety by the preliminary coordinates of its points and the linear constraints imposed on the corrections to those coordinates.

Extended Free Network Adjustment Constrains

Extended Free Net-Theoretical Background

Observation equations: $\ell + \mathbf{v} = \mathrm{C}\mathbf{w}$

The vector **w** is partitioned into global and local components through the introduction of a vector of parameters **y**.
y represents contribution of the measurements to the global component of the coordinates.
x is a vector of sterilized coordinates.

The elements of **y** can be relate as the parameters of a transformation between two vectors **x** and **w**.

Therefore **w** can be presents as: $\mathbf{w} = \mathbf{D}\mathbf{x} + \mathbf{F}\mathbf{y}$

$\mathbf{D} = \partial\mathbf{w}/\partial\mathbf{x}$ is a u by u full rank matrix-deformation matrix.

$\mathbf{F} = \partial\mathbf{w}/\partial\mathbf{y}$ is a u by f full column rank matrix.

Extended Free Net-Theoretical Background

New observation equations:

$$\ell + \mathbf{v} = \mathbf{C}(\mathbf{Dx} + \mathbf{Fy}) = \mathbf{C}(\mathbf{D}, \mathbf{F})\begin{bmatrix}\mathbf{x}\\ \mathbf{y}\end{bmatrix} = (\mathbf{A}, \mathbf{B})\begin{bmatrix}\mathbf{x}\\ \mathbf{y}\end{bmatrix}$$

When **D=I** and **F=0**, we have **x=w** and therefore:

$$\ell + \mathbf{v} = \mathbf{Ax}$$

the widespread model of a network adjustment

Similarity Transformation

It is used to transform one solution, **x**, pertaining to a certain datum into another solution, $\bar{\mathbf{x}}$, pertaining to another datum.

The transformation is described by:

$$\bar{\mathbf{x}} = \mathbf{x} + \mathbf{Ep}$$

E - Helmert's transformation matrix.
p - vector of d datum transformation parameters.

For a unique solution of $\bar{\mathbf{x}}$ that yields $\bar{\mathbf{x}}^{\mathrm{T}}\mathbf{P}_{\mathrm{x}}\bar{\mathbf{x}} \rightarrow min$

$$\mathbf{p} = -(\mathbf{E}^{\mathrm{T}}\mathbf{E})^{-1}\mathbf{E}^{\mathrm{T}}\mathrm{P}_{\mathrm{x}}\mathbf{x}$$

The transformation receives the form:

$$\bar{\mathbf{x}} = [\mathbf{I} - \mathbf{E}(\mathbf{E}^{\mathrm{T}}\mathbf{P}_{\mathrm{x}}\mathbf{E})^{-1}\mathbf{E}^{\mathrm{T}}\mathbf{P}_{\mathrm{x}}]\mathbf{x} = \mathbf{Jx}$$

$$\bar{\mathbf{Q}} = \mathbf{JQJ}^{\mathrm{T}}$$

Extended Similarity Transformation

The transformation from one solution of **x** and **y**, pertaining to a certain datum, into another $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$, pertaining to another datum, is obtained as follows:

$$\bar{\mathbf{x}} = \mathbf{x} + \mathbf{D}^{-1}\mathbf{E}\mathbf{p} + \mathbf{R}\mathbf{q}$$
$$\bar{\mathbf{y}} = \mathbf{y} + \mathbf{q}$$

q - vector of f variations in the **y** parameters.

$\mathbf{R} = -\mathbf{D}^{-1}\mathbf{F}$ - u by f full column rank matrix, **R** represents an apparent functional relationship between **x** and **y**.

For a unique solution of $\bar{\mathbf{x}}$ that yields $\bar{\mathbf{x}}^{T}\mathbf{P}_{x}\bar{\mathbf{x}} \rightarrow min$
it is required that $\partial(\bar{\mathbf{x}}^{T}\mathbf{P}_{x}\bar{\mathbf{x}})/\partial\mathbf{p} = 0$ and $\partial(\bar{\mathbf{x}}^{T}\mathbf{P}_{x}\bar{\mathbf{x}})/\partial\mathbf{q} = 0$

Extended Similarity Transformation

$$\begin{bmatrix} \mathbf{E}^{T}(\mathbf{D}^{-1})^{T}\mathbf{P}_{x}\mathbf{D}^{-1}\mathbf{E} & \mathbf{E}^{T}(\mathbf{D}^{-1})^{T}\mathbf{P}_{x}\mathbf{R} \\ \mathbf{R}^{T}\mathbf{P}_{x}\mathbf{D}^{-1}\mathbf{E} & \mathbf{R}^{T}\mathbf{P}_{x}\mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix} = - \begin{bmatrix} \mathbf{E}^{T}(\mathbf{D}^{-1})^{T}\mathbf{P}_{x}\mathbf{x} \\ \mathbf{R}^{T}\mathbf{P}_{x}\mathbf{x} \end{bmatrix}$$

$$\bar{\mathbf{x}} = \mathbf{x} + \mathbf{D}^{-1}\mathbf{E}\mathbf{p} + \mathbf{R}\mathbf{q} = \mathbf{x} + \begin{bmatrix} \mathbf{D}^{-1}\mathbf{E} & \mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix}$$

$$\bar{\mathbf{x}} = \left[\mathbf{I} - \begin{bmatrix} \mathbf{D}^{-1}\mathbf{E} & \mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{E}^{T}(\mathbf{D}^{-1})^{T}\mathbf{P}_{x}\mathbf{D}^{-1}\mathbf{E} & \mathbf{E}^{T}(\mathbf{D}^{-1})^{T}\mathbf{P}_{x}\mathbf{R} \\ \mathbf{R}^{T}\mathbf{P}_{x}\mathbf{D}^{-1}\mathbf{E} & \mathbf{R}^{T}\mathbf{P}_{x}\mathbf{R} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{E}^{T}(\mathbf{D}^{-1})^{T}\mathbf{P}_{x} \\ \mathbf{R}^{T}\mathbf{P}_{x} \end{bmatrix} \right] \mathbf{x} = \mathbf{J}_{ex}\mathbf{x}$$

$$\bar{\mathbf{Q}} = \mathbf{J}_{ex}\mathbf{Q}\mathbf{J}_{ex}^{T}$$

Extended Free Net Adjustment and GPS

GPS vectors are used to determine the relative positions of the network points and the network datum parameters of **orientation** and **scale**.

Transformation of a single point from measured GPS coordinates, which contain the datum parameter, to coordinates which are stripped from their datum content is done by three rotations and a scale factor as follows:

$$\begin{pmatrix} x_w \\ y_w \\ z_w \end{pmatrix}_i = \begin{pmatrix} s & r_z & -r_y \\ -r_z & s & r_x \\ r_y & -r_x & s \end{pmatrix} \begin{pmatrix} x_x \\ y_x \\ z_x \end{pmatrix}_i = \mathbf{D} \begin{pmatrix} x_x \\ y_x \\ z_x \end{pmatrix}_i$$

Extended Free Net Adjustment and GPS

The transformation can also be presented in the following form:

$$\begin{pmatrix} x_w \\ y_w \\ z_w \end{pmatrix}_i = \begin{pmatrix} 0 & -z_x & y_x & x_x \\ z_x & 0 & -x_x & y_x \\ -y_x & x_x & 0 & z_x \end{pmatrix}_i \begin{pmatrix} r_x \\ r_y \\ r_z \\ s \end{pmatrix} = \mathbf{F}_i\mathbf{y}$$

Extended Free Network Solution

- By using the extended S-transformation the datum content of scale and orientations were stripped from the measured coordinates.
- The extended solutions of all campiness were calculated firstly relative to datum points that were set base on 24 sites which were measured in all of the four campaigns.
- The approximate coordinates that are used to create **F** were chosen as the adjusted coordinates of the 2010 measuring campaign.

Extended Parameters

Monitoring Epoch	Extended parameters - y			
	r_X [rad]	r_Y [rad]	r_Z [rad]	s [-]
1999.9425	-9.66E-8	-9.70E-9	1.95E-8	1-3.36E-8
2006.7096	-5.88E-8	-2.56E-9	3.14E-8	1-1.41E-8
2009.7423	-1.14E-8	3.60E-9	1.99E-9	1-3.14E-9
2010.7397	0	0	0	1-0

The extended parameters are an inevitable mixture between the deformation parameters and the datum components of the GPS vectors.

Two Steps Deformation Analysis

- ► The solution of the network points plane position for each monitoring epoch and their variance-covariance matrix (first step) are used as pseudo-measurements for movements analysis (second step).

- ► Statistical tests are applied for estimating the correspondence of the motion model and its significance.

Movements Analysis

A linear model of movements was tested to describe the plane position of the network points:

$$x_i = x_0 + \dot{x}\Delta t$$

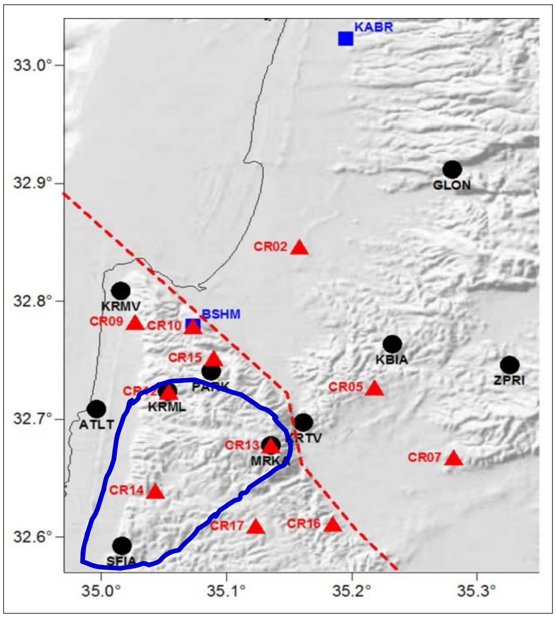
x_i - Position of a point in time t
t_0 - Reference epoch
$\dot{x}$ - Linear velocity
x_0 - Position at reference epoch
$\Delta t = t - t_0$

Weight Constrained Solution

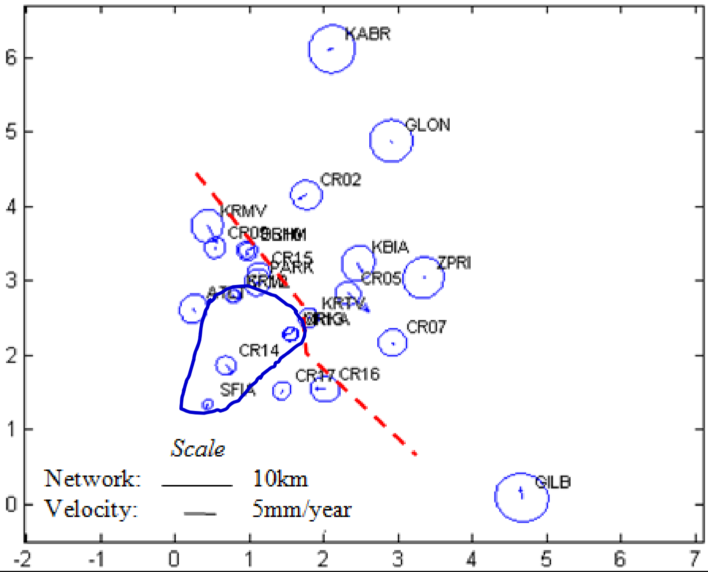
- ► The extended similarity transformation was applied to transform vector $\dot{x}$ and its covariance matrix to a weight constrained solution.
- ► Congruency testing was performed to determine the stable datum points.

Stable Datum Points

5 stable points in the center of the Carmel mountain region were selected to define the datum.



The Weight Extended Constraint Velocity Filed



Discussion and Conclusions

- Deformation parameters are estimated based on a time series of monitoring campaigns.
- For each monitoring campaign the measurements contribute in determining the relative positions of the network points and also in defining the network datum.
- GPS vectors contain part of the datum definition, such as scale and orientation.
- In monitoring deformation networks the part of the datum definition determined by the GPS vectors may not remain consistent.
- It is not possible to distinguish between the deformation parameters and the datum components of the vectors.

Discussion and Conclusions

- The measurements from each campaign are stripped from their datum content by Extended Similarity Transformation.
- The datum is defined by preliminary coordinates and linear constraints, which remain constant for all monitoring campaigns, as well as to define the position of the network points.
- The variations in the network geometry modeled by means of a linear model of movements.