Attitude Determination by Means of Dual Frequency GPS Receivers

Vadim ROKHLIN and Gilad EVEN-TZUR, Israel

Key words: GPS, Attitude determination, Dual frequency receiver

SUMMARY

In many civil and military systems direction finding with high accuracy is of top importance. The resolution of Attitude Determination (AD) can be performed by using Global Navigation Satellite System such as GPS. The concept of attitude determination by GPS is to initially calculate the baseline between the receiving antennas, by differential positioning, and then, calculate the attitude parameters. In order to achieve precise attitude determination one can use GPS carrier phase observations with correctly resolved integer cycle ambiguities. Solving the integer number and the attitude parameters is done via Least Squares Adjustment.

When data collected by single frequency receiver, the initializing of the attitude determination system takes up to 10 minutes with correctly resolved integer ambiguity. This research examines the influence of additional frequency L2 added to L1 and its impact on duration of initialization time and precision of the solution. In this article we illustrate that the use of a dual-frequency receiver significantly cuts back on the time required to find a solution, yet makes no dramatic contribution to the accuracy of the solution.

The speed of the IA solution depends on changes in the receiver-satellite geometry; so we implement an antenna rotation algorithm, in order to speed up the geometrical change. Results indicate that the use of rotation technique shortens the initialization process when gathering data from a single-frequency receiver; and has no such effect when collecting data from a dual-frequency receiver.

Attitude Determination by Means of Dual Frequencies GPS Receiver

Vadim ROKHLILN and Gilad EVEN-TZUR, Israel

1. INTRODUCTION

Attitude determination in air, sea or land with high accuracy and high reliability has always been a technological and engineering challenge. Direction finding information is required nowadays by many civilian systems like car navigation systems and military systems as unmanned aircrafts system. Some instruments such as a compass or a gyroscope are useful yet inherently limited. A magnetic compass determines direction relative to the magnetic north and not to the true north. It cannot be used in magnetically induced areas. A Gyroscope accumulates errors due to drifts, thus, requiring periodic recalibrations.
GPS allows for additional ways of Attitude Determination (AD). It operates 24 hour a day and is not prone to any of the mentioned limitations.

1.1 Integer ambiguity solution

The accuracy of position, speed and direction by GPS receiver depends on the quality of several important variables such as the quality of the pseudo range measurements, the carrier phase measurements, and the accuracy of the GPS satellite ephemeris. Accurate direction finding requires measuring the transmitted wave phases, and not just the pseudo range (PR). Many methods were designed to handle the issue of phase measurements, and solving for the Integer Ambiguity (IA). Many IA algorithms employ "On-The-Fly" (OTF) techniques, such as the Ambiguity Function Method (AFM) (Remondi, 1984), Least Squares Ambiguity Search Technique (LSAST) (Hatch, 1990), Fast Ambiguity Resolution Approach (FARA) (Frei and Beutler, 1990), and Least-Square Ambiguity Decorrelation Adjustment (LAMBDA) (Teunissen, 1994). AFM technique uses only the non-integer part of the phase measurement, and therefore the integer part is not affected by cycle slips. However, it requires increased measurement in order to receive an accurate solution.
The implemented techniques are performed in three stages: A float solution; evaluation of integer values by a search method; and a final fix solution. All these methods use of the variance-covariance matrix, obtained during the float solution stage. The methods differ in the ways the IA values are evaluated.
There are many applications which require an accurate real-time solution using kinematic or position differences methods. Therefore integer ambiguities must be solved in the shortest time. The objective is to obtain correct integer values within a few seconds. As long as the IA values have not been solved, one cannot obtain accuracy of position up to the level of centimeters. The conditions under which IA values can be solved depend not only on sophisticated calculation methods, but mostly on physical system parameters such as distance between antennas, number of visible satellites, multipath errors, and other errors which could not necessarily be modeled. These factors have an impact on the quality and accuracy of the measurements.

1.2 Attitude Determination

The basic principle of attitude determination (AD) by GPS is based on interferometry methods, using at least two antennas. The GPS signal propagates at the speed of light, arriving to the antenna closest to the satellite, earlier than it to other antennas. By calculating the phase differences between pairs of antennas, using several satellites and three or more antennas, the relative distance between the antennas can be determined. The same means can be used to calculate Euler angles as well (attitude parameters, roll, pitch and heading). Heading and pitch can be determined using only two antennas (Kaplan et.al, 2006).
Therefore, calculating the attitude parameters requires first to calculate the vector between the antennas using differential solution techniques, and only then calculate the attitude parameters. Calculating the vector can be done by pseudo range or by phase measurements. Accurate applications require using phase measurements and solving IA (Dai et.al, 2008).
We have developed a unique and highly accurate solution for attitude determination algorithm based on single frequency receivers. This algorithm uses two antennas that are fixed to a rigid platform and located less than one meter apart. The algorithm is based on two modes: initialization and normal. During initialization mode GPS signals are received from a satellite, then IA values have been solved and they are used to calculate the baseline vector and the attitude parameters of the platform. During the normal mode the algorithm provides the attitude parameters continuously for every epoch by kinematic measurements. The initialization process that solves the observation equations and finds the integer ambiguity takes more than 10 minutes. This time is too long for GPS AD systems to operate as real-time systems. The main purpose of the current research is to minimize initialization time up to several minutes. For this reason, dual frequency GPS receivers is used for a faster solution of the initialization mode, and examination of two frequencies together and their combination on the solution of the attitude parameters.
The calculation time of the integer number of the wavelengths depends on geometrical changes between the receiver and the satellite positions; therefore this study implements the antenna rotation technique (Tu et.al, 1996).

2. DIFFERENTIAL GPS SOLUTION

GPS observations contain mostly code and phase measurements. Compared to code measurements, the main advantage of phase measurements is a low error level, usually less than 1 mm with lower effect of multipath errors (Leick, 1995). And so, phase measurements are an important element of accurate attitude determination systems.
Phase equations for both L_1 and L_2 frequencies can be modeled as:

$$\begin{aligned} \Phi^k_{i,L_1}(t) &= \lambda_{L_1} \cdot \phi^k_{i,L_1/L_2}(t) = \rho^k_i + c\left[dt_i(t) - dt^k(t)\right] - I^k_{i,L_1} + T^k_i + \lambda_{L_1} N^k_{i,L_1} - \varepsilon^k_{i,L_1} \\ \Phi^k_{i,L_2}(t) &= \lambda_{L_2} \cdot \phi^k_{i,L_1/L_2}(t) = \rho^k_i + c\left[dt_i(t) - dt^k(t)\right] - I^k_{i,L_2} + T^k_i + \lambda_{L_2} N^k_{i,L_2} - \varepsilon^k_{i,L_2} \end{aligned} \quad (1)$$

$\Phi^k_{i,L_1/L_2}(t)$ represents phase measurements between satellite k and receiver i, on L_1 or L_2 frequencies in meter. ρ^k_i represents the true distance between the satellite and the receiver. c is the speed of light. $dt_i(t)$ and $dt^k(t)$ are clock errors of the receiver and the satellite respectively. $I^k_{i,L_1/L_2}$ is the ionosphere delay in meter, and T^k_i is the troposphere delay in meter.

λ_{L_1/L_2} represents the phase wavelength. N_{L_1/L_2} represents the integer number of the wavelengths for L_1 and L_2 frequencies respectively. $\varepsilon^{k}_{i,L_1/L_2}$ represents additional noise errors for each input, such as phase center movements of the antenna, multipath effects, electronic equipment errors, and other factor which could not be modeled.

Since the vector between antennas of attitude determination systems is very short, ionosphere and troposphere errors are assumed to affect both receivers in a similar way (Teunissen 1998), and can thus be canceled out (Leick, 1995).

Since the measurements in equation (1) are affected by atmospheric and hardware errors (receiver and satellite), one of the effective ways to decrease and eliminate their influence is to calculate measurements differences. When two receivers measure phase from same satellite at the same time, satellite and receiver clock error are cancelled out, and satellite orbital errors and atmospheric delays of the GPS signal are significantly minimized (Hofmann *et al.* 1997). This process is known as double difference - DD (∇Δ):

$$\begin{aligned}\nabla\Delta\Phi^{i,j}_{AB,L_1} &= \nabla\Delta\rho^{i,j}_{AB} + \lambda\nabla\Delta N^{i,j}_{AB,L_1} + \nabla\Delta\varepsilon^{i,j}_{AB} \\ \nabla\Delta\Phi^{i,j}_{AB,L_2} &= \nabla\Delta\rho^{i,j}_{AB} + \lambda\nabla\Delta N^{i,j}_{AB,L_2} + \nabla\Delta\varepsilon^{i,j}_{AB}\end{aligned} \tag{2}$$

Although $\nabla\Delta\Phi^{i,j}_{AB,L_1/L_2}$ values can be obtained via GPS receiver measurements the IA values of phase measurements are still unknown. In order to handle the DD measurements and solve the IA values, two approaches are applied in this study. Both approaches include differential measurements use dual frequency receivers with a less than one meter short baseline. The first approach is with static measurements. Using this approach, we tested the effect of adding an L_2 frequency to evaluate the solution time of the observation equations, and found the resulting integer ambiguity. When using L_1 frequency, the minimal required measurement time for initial solution, based on previous experiments, was about 100 seconds. This time was reduced to 40 seconds when L_1 and L_2 frequencies were used. . The second approach makes use of the baseline rotation technique, which we will be discussed later on.

Both methods, the baseline rotation method and static measurement, solve the vector and the number of wavelengths, which at this stage are real and not integer numbers. The LAMBDA algorithm is used in order to obtain integer values (Teunissen 1998). This algorithm receives non-integer ambiguities values and their cofactor matrix, and solves the minimization problem according to the Least Squares Adjustment:

$$\min_{a,b}\left\| y - A_{\lambda}a - A_{\alpha}b \right\|^{2}_{Q_y} \quad ,a\in Z^{n}, b\in R^{p} \tag{3}$$

y is the measurement vector, *a* represents the IA value vector. A_{λ} is the coefficient matrix of the IA values, *b* represents the baseline vector , and A_{a} is the coefficient matrix of the baseline vector.

3. LINEAR COMBINATION OF THE FREQUENCIES

GPS satellites transmit both L_1 and L_2 frequencies. A combination of these frequencies can contribute to a faster solution of the IA values. Linear combinations are mainly used to minimize or to cancel out the influence of different factors such as atmospheric delays. Common combinations are wide lane (L_W), narrow lane (L_N) and Ionosphere-free (L_C) (Hofmann *et al.* 1997). Equation (4) presents the possible combinations in the DD equations of phase measurements for $L1$ and L_2 frequencies, when a_1 and a_2 are the combination coefficients:

$$\nabla\Delta\Phi^{ij}_{AB[a1,a2]} = a_1 \cdot \nabla\Delta\Phi^{ij}_{AB[L1]} + a_2 \cdot \nabla\Delta\Phi^{ij}_{AB[L2]} \qquad (4)$$

Table 1 presents the wavelengths and coefficients necessary to create the frequencies combination. Using the Ionosphere-free combination significantly minimizes the effects of Ionosphere delays (Leick, 1995). Since this method returns the $a_2 = f_{L2} / f_{L1}$ parameter as a non-integer number, it makes the calculation of the IA values as integers very hard. In this study the length of the baseline between the two antennas is approximately one meter. Since the wavelength of the L_W combination (86 cm) is very close to the length of the baseline, it allows a faster convergence of the solution to the correct IA values. The L_W combination significant drawback is its highest noise level relative to the original frequencies, which can cause a noisy and inaccurate solution. However, the L_N combination has a shorter wavelength (10.7 cm), which leads to a longer convergence, but lower noise levels when compared to the L_1 and L_2 frequencies, which ensure a more accurate solution.

frequency	a_1 parameter	a_2 parameter	wavelength[cm]	notes
L1	1	0	19.0	Original L_1 frequency
L2	0	1	24.4	Original L_2 frequency
L_C	1	f_{L2}/f_{L1}	48.4	Ionosphere free
L_W	1	-1	86.2	Wide Lane
L_N	1	1	10.7	Narrow Lane

Table 1 – The most commonly used Linear Combinations when working with the original L_1 and L_2 frequencies.

4. THE BASELINE ROTATION METHOD

The baseline rotation method is mainly used to force a quicker geometrical change between the satellites and the receivers and to shorten the solution time for the integer ambiguity. Assuming that at t_1 receiver R_1 is connected to antenna A and is located above station 1, and receiver R_2 is connected to antenna B and is located above station 2 (see left side of figure 1). The antennas are swapped between the stations at t_2, antenna A is switched to station 2 and antenna B is switched to station 1(see right side of figure 1). This antenna swap method is equivalent to a 180º angle rotation of attitude determination array of a rigid rod with two antennas. Position I is defined as the antennas alignment before the swap, and position II is their alignment after it. $(dx_{AB}, dy_{AB}, dz_{AB})_I$ represents the vector between both antennas before the rotation. The vector after the 180º rotation is equivalent to the vector before the rotation with a reversed sign $(dx_{AB}, dy_{AB}, dz_{AB})_{II} = (\text{-}dx_{AB}, \text{-}dy_{AB}, \text{-}dz_{AB})_I$.

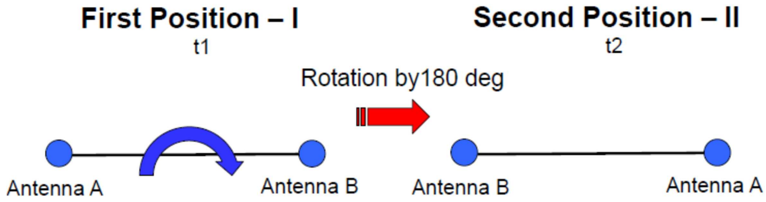


Figure 1 – The rotation of the baseline at a 180º angle.

The observation equations of the double differences of the phase measurement before and after the rotation for all 4 satellites at a single epoch are:

$$l^{DD}_{L_1/L_2,I} = A_{a_I}\begin{pmatrix} dx_{AB} \\ dy_{AB} \\ dz_{AB} \end{pmatrix}_I + A_{\lambda_{L_1/L_2}}\begin{pmatrix} N^{jk}_{AB,L_1} \\ N^{jl}_{AB,L_1} \\ N^{jm}_{AB,L_1} \\ N^{jk}_{AB,L_2} \\ N^{jl}_{AB,L_2} \\ N^{jm}_{AB,L_2} \end{pmatrix}; \quad l^{DD}_{L_1/L_2,II} = A_{a_{II}}\begin{pmatrix} dx_{AB} \\ dy_{AB} \\ dz_{AB} \end{pmatrix}_{II} + A_{\lambda_{L_1/L_2}}\begin{pmatrix} N^{jk}_{AB,L_1} \\ N^{jl}_{AB,L_1} \\ N^{jm}_{AB,L_1} \\ N^{jk}_{AB,L_2} \\ N^{jl}_{AB,L_2} \\ N^{jm}_{AB,L_2} \end{pmatrix} \tag{5}$$

Vector $l^{DD}_{L_1/L_2,I/II}$ represents the double difference carrier phase measurements_for positions I and II. $A_{a_{I/II}}$ is a coefficient matrix of the vector between the antennas, before and after the rotation and $A_{\lambda_{L_1/L_2}}$ represents a coefficient matrix of the L_1 and L_2 wavelengths. It is assumed that there are no cycle slips between epochs during the calculations of the IA parameters, keeping the $A_{\lambda_{L_1/L_2}}$ coefficient matrix fixed before and after the antenna rotation. Reducing the observation equations (5) before and after the rotation allows the calculation of the vector only due to the reduction of the IA parameters.

$$l^{DD}_{L_1/L_2,II} - l^{DD}_{L_1/L_2,I} = -(A_{aII} + A_{aI})\begin{pmatrix} dx_{AB} \\ dy_{AB} \\ dz_{AB} \end{pmatrix} \tag{6}$$

A 180º rotation is a special case of the baseline rotation. Figure 2 depicts the rotating of the antenna array at a non 180º angle.

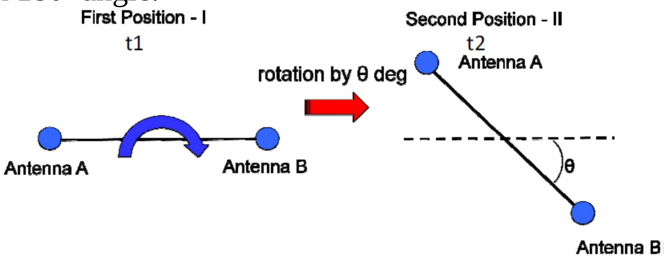


Figure 2 - Rotating the baseline at a non-180º angle.

θ is the rotation angle of the antenna array. $R_3(\theta)$ represents a rotation matrix around the *u* axis. $R_3(\theta)$ function is to rotate the vector in the local level coordinate frame (LLF) around the vertical axis at θ angle. That is why the vector values in LLF system in position I can be rotated into position II by using the equation,

$$\begin{pmatrix} de \\ dn \\ du \end{pmatrix}_{\mathrm{II}} = R_3(\theta)\begin{pmatrix} de \\ dn \\ du \end{pmatrix}_{\mathrm{I}} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} de \\ dn \\ du \end{pmatrix}_{\mathrm{I}} \tag{7}$$

The vector in LLF coordinate system can be calculated from the Cartesian system vector by using the $\tilde{P}$ matrix. Matrix $\tilde{P}$ is rotation function of the geographical coordinates φ, λ, h of one of the edge points of the vector ($\tilde{P}$ is an orthogonal matrix, meaning $\tilde{P}^{-1} = \tilde{P}^{t}$).

$$\tilde{P} = \begin{pmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\varphi\cos\lambda & -\sin\varphi\sin\lambda & \cos\varphi \\ \cos\varphi\cos\lambda & \cos\varphi\sin\varphi & \sin\varphi \end{pmatrix} \tag{8}$$

The relation between the vector in position I in a Cartesian system, and that obtained in position II is:

$$\left(dx_{AB} \quad dy_{AB} \quad dz_{AB}\right)_{\mathrm{I}}^{T} = \tilde{P}_{\mathrm{I}}^{-1} R(\theta)^{T} \tilde{P}_{\mathrm{II}} \left(dx_{AB} \quad dy_{AB} \quad dz_{AB}\right)_{\mathrm{II}}^{T} \tag{9}$$

By using equation (9) in equation (5), and subtracting the observation equations before and after the rotation, only the vector can be calculated without solving for the IA variables.

$$l_{II} - l_{I} = \left(A_{a_{II}} - A_{a_{I}} \tilde{P}_{\mathrm{I}}^{-1} R(\theta)^{T} \tilde{P}_{\mathrm{II}}\right) \begin{pmatrix} dx_{AB} \\ dy_{AB} \\ dz_{AB} \end{pmatrix}_{II} \tag{10}$$

Since we are interested not only in the vector, but also in the IA values, we can calculate them using equation (5), by inserting known vector values (solved by equation (6) or (10)), and solving only for the IA values. If we add the equations appearing in (5) (instead of subtraction as we did before), we get equation (11), requiring a simultaneous solution of the vector components and the IA values. However, the quick geometrical change in position relative to the satellite is expressed in equation (11).

$$l_{II} + l_{I} = \left(A_{a_{II}} + A_{a_{I}} \tilde{P}_{\mathrm{I}}^{-1} R(\theta)^{T} \tilde{P}_{\mathrm{II}}\right) \begin{pmatrix} dx_{AB} \\ dy_{AB} \\ dz_{AB} \end{pmatrix}_{II} + 2A_{\lambda} \begin{pmatrix} N_{AB}^{jk} \\ N_{AB}^{jl} \\ N_{AB}^{jm} \end{pmatrix} \tag{11}$$

5. RECURSIVE LEAST SQUARE ADJUSTMENT

One of the major of the research was to obtain a faster correct solution to the IA variables and the baseline vector. When we use measurements from GPS receivers, there is always a possibility of inaccurate solution while calculating the variables with small number of measurements. Multipath errors or a poor geometrical distribution of the satellites could affect the solution. Thus, we may not always get correct solution with minimal measurements of IA variables and the baseline vector, as was discussed in chapter 2. In such a case the algorithm remains in the initialization mode and then it proceeds to recursive least square adjustment method. The system continues to add measurements and solve IA variables and baseline vector until the length of the calculated vector will equal the given vector length. The given length between the phase centers of the antennas is measured at the beginning of the process in very accurately and is used as the basis for comparison. Once the difference between the calculated length of the vector and the known length is less than a threshold, the initialization mode is complete and the algorithm advances to the normal mode.

The recursive least square method adds new measurements after every new epoch. These measurements are included into the least square along with the previous solution, while taking into account the cofactor matrix of the previous solution (Mikhail, 1976), achieving more accurate value of variables.

The solution after the addition of a new epoch d_i of the vector and the IA values based on the old solution d_{i-1} is equal to:

$$d_i = d_{i-1} - N_{i-1}^{-1} C_i^t \left(I + C_i N_{i-1}^{-1} C_i^t \right)^{-1} \left(C_i d_{i-1} - L_i \right) \tag{12}$$

N_{i-1}^{-1} is the cofactor matrix of the previous solution. C_i represents the coefficient matrix of the variables, and L_i represents the new DD measurements. The new cofactor matrix N_i^{-1} can be calculated by adding new measurements over the basis of the old N_{i-1}^{-1} cofactor matrix like so:

$$N_i^{-1} = N_{i-1}^{-1} \left(I - C_i^t \left(I + C_i N_{i-1}^{-1} C_i^t \right)^{-1} C_i N_{i-1}^{-1} \right) \tag{13}$$

After each epoch, the IA variables receive a more accurate float solution. At this stage the LAMBDA algorithm is employed in order to calculate the integer values of the IA. Using the updated IA values, the baseline vector is recalculated.

6. CALCULATING THE ATTITUDE PARAMETERS

The attitude determination (AD) parameters represent the direction of a coordinate system attached to platform relative to the Local Level Frame system (LLF) (Lu et.al, 1993). In AD systems based on GPS measurements, the coordinate system in LLF is as follows: the *u* axis points upwards in the direction of the ellipsoid's normal, the *e* axis points towards ellipsoidal east and the *n* axis points to the north. Since three points define a plane, a minimum of three antennas define the Antenna Body Frame (ABF) coordinate system. The ABF origin is selected to coincide with the phase center position of one of the antennas, predefined as the master antenna and all other antennas are referred to as slaves. We assume the antennas are fixed to a rigid platform, and their relative spatial configuration is fixed. Positive *y* axis is defined on the line from antenna 1 to antenna 2. The *x* axis is orthogonal to the *y* axis and positioned in a plane defined by antennas 1, 2 and 3. The *z* axis is orthogonal to both *x* and *y*, pointing upwards. The LLF coordinate system can be adjusted to the ABF system by combining three continuous rotations around the three axes of the Local Level coordinate system. First rotation is around *u* axis, referred to as heading (*h*). Second rotation is around *n* axis, referred to as pitch (*p*). Third rotation is around *e* axis, referred to as roll (*r*). $R(h, p, r)$ is an orthogonal rotation matrix that rotates a GPS vector from the LLF system $(n, e, u)^T$ to the ABF system $(x^b, y^b, z^b)^T$. The rotation matrix R is equal to $R_2(r) \cdot R_1(p) \cdot R_3(h)$ when R_1, R_2 and R_3 are rotation matrixes relative to the *n*, *e* and *u* axes respectively.

$$\begin{pmatrix} x^b \\ y^b \\ z^b \end{pmatrix} = R(h, p, r) \cdot \begin{pmatrix} n \\ e \\ u \end{pmatrix} \tag{14}$$

The AD parameters can be found directly, or by using least square attitude estimation (LSAE). The direct approach is quick and simple and uses only three antennas in order to solve the AD parameters. When a system has more than three antennas, the calculation is based on three antennas only. However, the LSAE approach makes use of all available information during the calculation of the AD parameters. The more antennas are factored in the calculation, the more accurate and reliable the results will be (Dai et.al, 2008).

6.1 The direct approach

Using the direct approach, the attitude parameters can be calculated using only a local level coordinate system, with no prior knowledge of the coordinates in the antenna body system (Lu et.al, 1993).

Based on the ABF definition, b_{12} represents a baseline between antennas 1 and 2, and the coordinates of antennas 2 and 3 in the ABF system can be expressed as $b_2 = \left(0, b_{12}, 0\right)^T$ and $b_3 = \left(x_{3,b}, y_{3,b}, 0\right)^T$ respectively. After assigning the coordinates of antenna 2 into equation (14) in ABF system and rearranging the equation, we will receive:

$$
\begin{aligned}
h &= -\tan^{-1}\left(\frac{x_{2,l}}{y_{2,l}}\right), \\
p &= -\tan^{-1}\left(\frac{z_{2,l}}{\sqrt{x_{2,l}^2 + y_{2,l}^2}}\right)
\end{aligned}
\qquad (15)
$$

We see that only two antennas (1 and 2) are required in order to determine the pitch and heading of the platform. Once these parameters are known, we can rotate the coordinates of antenna 3 $l_3 = \left(x_{3,l}, y_{3,l}, z_{3,l}\right)$ in the local level system around the pitch and heading angles. The coordinates need to be transformed afterwards to the ABF system in order to calculate the roll parameter *r*:

$$
r = -\tan^{-1}\frac{z'_{3,l}}{x'_{3,l}} \qquad (16)
$$

7. IMPLEMENTATION METHODS

The program that was developed during this study is based on the Matlab code of Dai, Knedlik, and Loffeld, developed in 2008 in the Siegen University is Germany (Dai et.al, 2008). Originally the program solved only data from code measurements supplied by a single-frequency GPS receiver, and measured in a static state only. The added program has capabilities to receive data measurements of a dual-frequency GPS receiver and solve the IA and vector components based on L_1 and L_2 frequencies and the frequency combinations such as L_W, L_N, L_1+L_W, L_1+L_N, as disbudded in chapter 4. The ability to solve the IA values and the vector have been added using the baseline rotation method along with a recursive solution of variables in the case that the obtained solution is not based on a minimal number of measurements, as mentioned in chapter 5.

In order to test and check the program, several experiments were performed using a simple device that allows testing in static and controlled rotation states (Figure 3). The device is constructed of two main parts: a pointer and a sensor. The pointer is made out of a rigid aluminum rod that indicates the direction of the baseline with two antennas. The rod is attached to a flat plate allowing a rotation of the rod at 45° intervals. The sensor is made up of two dual-frequency GPS Ashtech Z-Surveyor receivers connected to two AeroAntenna antennas, model AT2775. Each antenna is positioned on metal plate in order to minimize multipath errors. Rotation of the rode with the antennas to the desired angle is preformed manually. The array is activated by a two modes process: initialization and normal. In the initialization mode the rod is rotated to the desired degree in order to solve the IA. During the normal mode azimuth measurements can be performed by rotating the pointer in the desired direction.

Figure 3: A direction finder constructed of two pieces: a pointer and a sensor.

The program was written in Matlab software and the system performance cannot be tested in real time. During initialization mode, the algorithm receives RINEX files sent from the receivers at the end of the measurement. The solution's steps of the IA and the vector are described in Figure 4. At the end of the initialization mode, the algorithm enters into the normal mode.

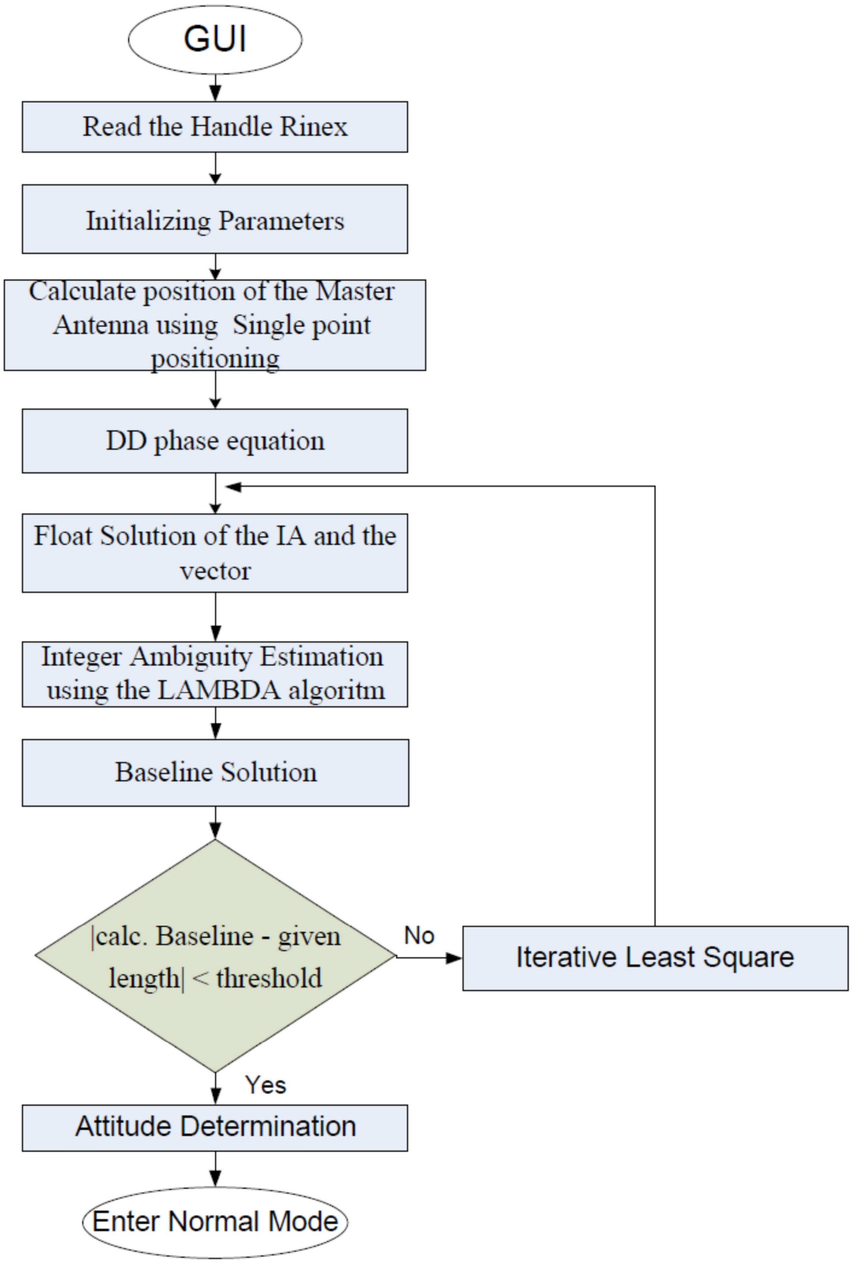


Figure 4 – Diagram of the developed algorithm

8. RESULTS

The results of the performed test are a good illustration of the developed system abilities. Dual-frequency Ashtech Z-Surveyor receivers were used in this experiment. The experiment was conducted on 1.6.2011 for a span of 80 minutes during which data was collected at 1 second epoch intervals. 8 satellites were used for the solution of the observation equation, and the Pdop values ranged between 1.8 and 2.1.
Static measurements were performed during the first experiment (Fig. 5). This experiment tested the combined frequencies, L_2 frequency measurements were added to the L_1 frequency.

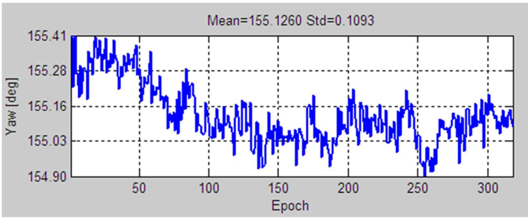
Other combinations of frequencies and their contribution to the initialization time and the accuracy of the solution were also evaluated. This experiment was divided into six stages (a to f). During stage (a) the unknowns, IA and the vector, were solved by using L_1 measurements only. During stage (b) the unknowns were solved using L_1 and L_2 measurements. During stages (c) and (d) the unknowns were solved according to frequencies combinations meant to produce L_W and L_N, and during stages (e) and (f) the combination of L_1 frequency and the L_W and L_N combinations were tested.

- The results of (a) and (b) experiments (Fig 5) show that the addition of the L_2 frequency to the L_1 frequency shortens significantly the initialization time. When using L_1 frequency only, the time to solve the integer number of the wavelengths is a little over 3 minutes, and the time to solve the IA when using L_1 and L_2 frequencies is 40 seconds. Both experiments produced similar standard deviations.
- The solution time for the IA using L_W combination (c) is 100 seconds. This time is significantly shorter than the solution time while using only the L_1 frequency (a). The standard deviation in this experiment is 0.59º, significantly bigger relative to experiment (a).
- The solution time of the IA using L_W combination (d) is 225 seconds. This time is close to the solution time when using only L_1 frequency (a). The standard deviation in this experiment is 0.07º, significantly smaller relative to experiment (e) and similar to experiment (a).
- According to (e) and (f) results, the IA are solved in 20 seconds when using a combination of L_1 and L_W frequencies, and 80 seconds when using the frequencies combination L_1 and L_N. Thus, combining $L1$ and L_W with L_1 and L_N significantly shortens the solution time for the integer number of the wavelengths. The standard deviation of experiment (e) is 0.33º.

The second experiment (Fig 6) was conducted according to the baseline rotation method. This experiment tested the influence of the rotation on the duration of the initialization. The experiment contained two stages, during stage (a) the unknowns were solved according to the measurements of L_1 alone, and during stage (b) the unknowns were solved according to L_1 and L_2 measurements. During the experiment a 90º rotation was performed. Sections (a.1) and (b.1) of Fig 6 illustrate the calculation of the azimuth using the static measurement method before the rotation. Sections (a.2) and (b.2) of the Figure illustrate the calculation of the azimuth using the static measurement method after the rotation. The azimuth values received during stage (a) and (b), before and after the rotation, were calculated using the static measurement method. The discrepancy between the azimuth values before and after the rotation is due to the angular rotation that was made during the experiment. Sections (a.1), (a.2) and (b.1), (b.2) illustrate that the calculated rotation angle is equal to the actual rotation angle.

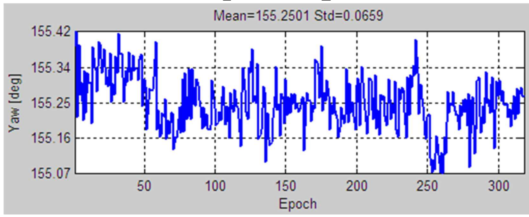
The solution time for the integer number of wavelengths when using the *L1* frequency alone (stage a) is 230 seconds. This time includes 50 seconds for rotating the pointer (baseline), and 90 seconds of information gathering in a static state before and after the pointer rotation (90+50+90=230s). The solution time when using the *L1* and *L2* frequencies (stage b) lasted a total of 70 seconds, 30 seconds of rotating the pointer, and 20 seconds of data gathering is the static state before and after the rotation of the pointer (20+30+30=70s). No significant contribution to the minimization of the solution time was detected using this method when the antennas are in the static state only.

(a) **L_1**



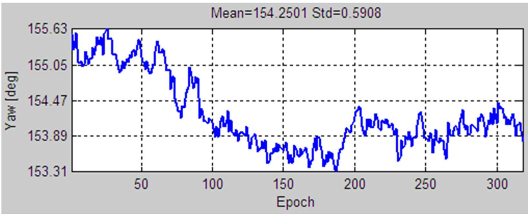
Mean = 155.12º
Std = 0.11º
Number of epochs = 200

(b) **L_1 and L_2**



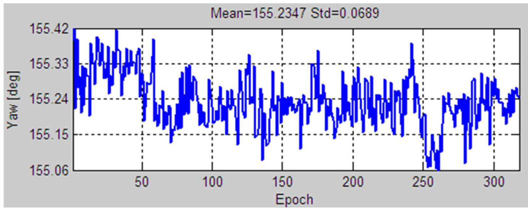
Mean = 155. 25º
Std = 0.07º
Number of epochs = 40

(c) **Lw**



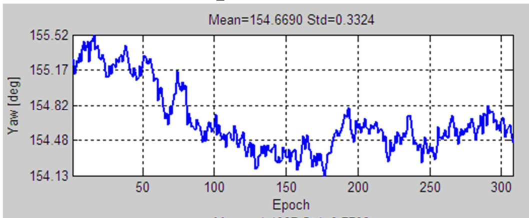
Mean = 154.25º
Std = 0.6º
Number of epochs = 100

(d) **Ln**



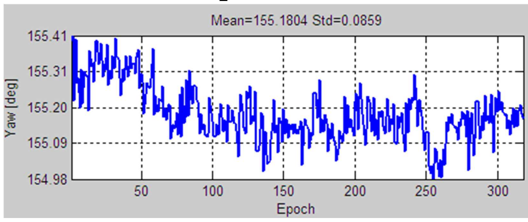
Mean = 155.23º
Std = 0.07º
Number of epochs =225

(e) **L_1 and Lw**



Mean = 154.67º
Std = 0.33º
Number of epochs = 20

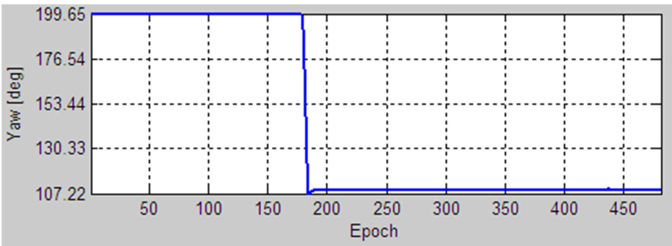
(f) **L_1 and Ln**



Mean = 155.18º
Std = 0.09º
Number of epochs = 80

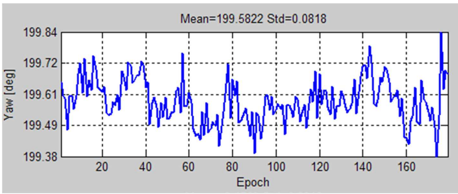
Figure 5 – Experiment results for the antennas in the static state. Sections (a) through (f) show azimuth calculation by frequency type:
(a) L_1, (b) L_1 and L_2, (c) L_W (d) L_N, (e) L_1 and L_W, (d) L_1 and L_N.

(a) **L_1**



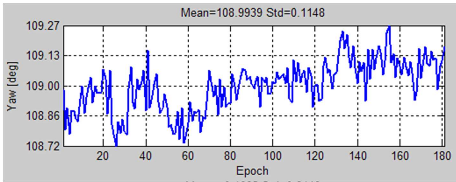
Number of epochs = 90+50+90 = 230 sec

(a.1)



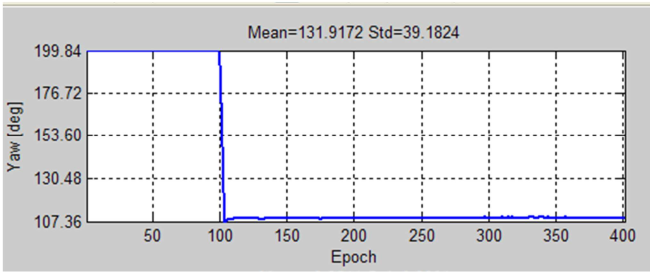
Mean = 199.58º
Std = 0.08º

(a.2)



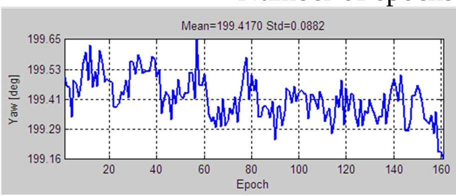
Mean = 108.99º
Std = 0.11º

(b) **L_1 and L_2**



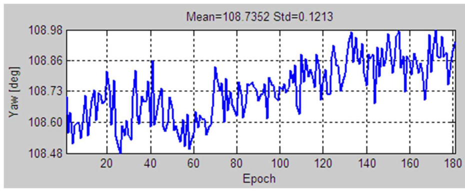
Number of epochs =20+30+20 = 70 sec

(b.1)



Mean = 199.42º
Std = 0.09º

(b.2)



Mean = 108.74º
Std = 0.12º

Figure 6: Experiment results using the baseline rotation method. (a) azimuth calculation using the L_1 frequency only. (a.1) azimuth calculation using the static measuring method before the rotation. (a.2) azimuth calculation using the static method after the rotation. (b) azimuth calculation using the L_1 and L_2 frequencies. (b.1) azimuth calculation using the static measuring method before the rotation. (b.2) azimuth calculation using the static measuring method after the rotation.

9. CONCLUSIONS

Combining different frequencies significantly shortens the solution time of the integer number of the wavelengths. An array of experiments have shown that in less than 30 seconds a solution can be obtained once the receiver starts picking up the satellites signals. These results depend on various factors such as the satellites geometrical alignment, the exposure of the receiver to multipath errors, the quality of the antennas and so forth.

The need for a quick solution depends upon the application. If the application requires a solution within a small timeframe, the user needs to purchase the right equipment, capable of receiving and processing more than one frequency simultaneously. The results show that *L1+L2* have no influence on the solution's accuracy and therefore, if the length of the initialization does not play a significant part, a GPS receiver can be used for the *L1* frequency only.

Theoretically, the baseline rotation method can be used to solve the integer number of the wavelengths in seconds. It is however important to remember that actual rotation requires time and resources, especially when using motorized rotation. Additionally, not all measurements are normally spaced because of many errors incurred during the data processing and the solution of the observation equations. Therefore, there are minimum measurements we need to perform in order to overcome the errors. The solution time of the obtained results using this process was not minimized by the rotation method used in the experiments.

REFERENCES

Dai, Z., Knedlik, S., Loffeld O., 2008, A MATLAB toolbox for attitude determination with GPS multi-antenna systems. Center for Sensor systems (ZESS), University of Siegen, 57068 Siegen, Germany

Hofmann-Wellenhof, B., Lichtenegger H., Collins J., 1997, GPS Theory and Practice, E. D. Kaplan and C. J. Hegarty , 2006, Understanding GPS Principles and Applications. 2nd edition. Artech House

Leick, A., 1995, GPS Satellite Surveying. 2nd edition. A Wily-interscience publication John Wiley & sonc, INC.

Lu, G., Cannon, M.E., Lachapelle, G., Kielland, P., 1993, Attitude Determination in a Survey Launch Using Multi-Antenna GPS Technologies. Department of Geomatics Engineering, The University of Calgary, Canadian Hydrographic Service. Proceeding of National Technical Meeting, ION, Alexandria VA, 251-160.

Mikhail, E. M., 1976. Observations and Least Squares. IEP-Dun-Donnelley Harper & Row, Publishers.

Teunissen P.J.G and Kleusberg, A., 1998,GPS for Geodesy. 2nd edition, Springer.

Tu, C.H., Tu, K.Y., Wang, L.S., 1996, GPS compass: a novel navigation equipment. IEEE Trans Aerosp Electr Syst 33(3):1063–1068.

BIOGRAPHICAL NOTES

Vadim Rokhlin is a master student of Mapping and Geo – Information Engineering at the Division of Mapping and Geo-Information Engineering, Technion – Israel Institute of Technology, Haifa, Israel.

CONTACTS

Vadim Rokhlin

Division of Mapping and Geo-Information Engineering,
Faculty of Civil and Environmental Engineering,
Technion City, Haifa 32000,
ISRAEL
Tel. +972 4 8292942
Email: vadimr@tx.technion.ac.il