

# On the Robustness of Next Generation Phase-only Real-Time Kinematic Positioning

Lennard Huisman  
Peter J.G. Teunissen  
Dennis Odijk

Curtin University of Technology  
GNSS Research Centre

## Motivation

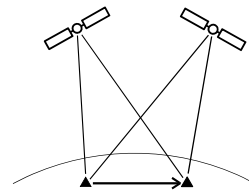
- Code observations more affected by multipath than phase observations
- Curiosity: Robustness and possibility next generation GNSS phase-only RTK

# Contents

- Real-Time Kinematic Positioning
- Phase-only versus Code and Phase
- Phase-only RTK design computations
- Phase-only RTK experimental results
- Conclusions

# Real-Time Kinematic Positioning

- Relative positioning
- Epoch-by-epoch solution
- Consider short baseline neglecting atmosphere



Resolving of double-differenced integer phase-ambiguities is the key to precise GNSS RTK positioning

## Integer Least Squares strategy

1. Standard LS adjustment gives real-valued float estimates for unknown (integer) ambiguity and baseline parameters
2. Resolve integer LS estimate for ambiguity parameters
3. Correct float baseline parameter to obtain fixed baseline solution

$$(i) \quad \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}, \quad \begin{bmatrix} Q_{\hat{a}\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}\hat{b}} \end{bmatrix}$$

$$(ii) \quad \check{a} = \arg \min_{a \in Z^n} \|\hat{a} - a\|_{Q_{\hat{a}\hat{a}}}^2$$

$$(iii) \quad \check{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}\hat{a}}^{-1} (\hat{a} - \check{a})$$

## Phase-only $\leftrightarrow$ Code and phase

- Code and phase RTK model

$$p_j(i) = \quad + G(i)b$$

$$\phi_j(i) = \lambda_j a_j + G(i)b$$

## Phase-only $\leftrightarrow$ Code and phase

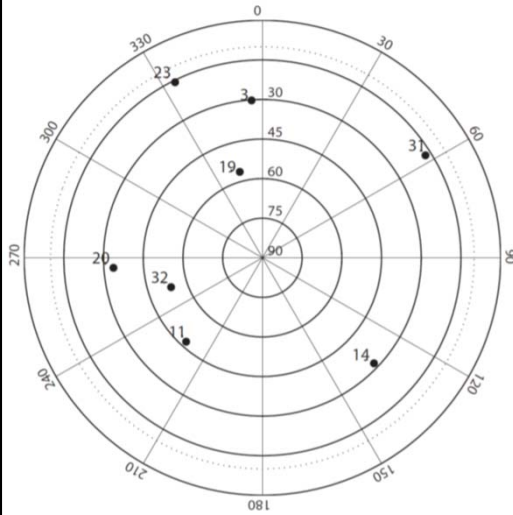
- Phase-only RTK model

$$\phi_j(i) = \lambda_j a_j + G(i)b$$

## Phase-only design computations

- Measure for performance: Bootstrapped ambiguity success-rate

## Phase-only design computations



- 8 satellites
- Elevation dependent weighting  

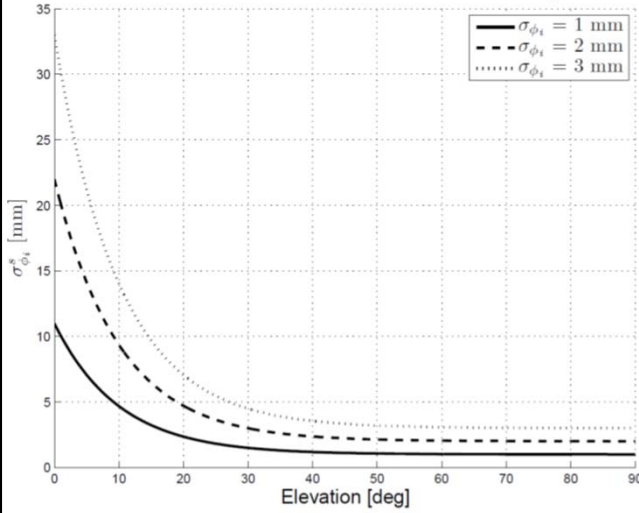
$$\sigma_{\phi_i}^s = \sigma_{\phi_i} (1 + 10 \exp\{-\frac{\delta_i^s}{10^\circ}\})$$
- Data rate 1Hz
- GPS L1/L2/L5 phase-observations

## GPS L1/L2/L5 phase-observations

Frequencies	Success-rate 4 satellites	Success-rate 5 satellites	Success-rate 6 satellites	Success-rate 7 satellites	Success-rate 8 satellites
L1	0.0000	0.0000	0.0002	0.0036	0.1602
L1/L2	0.2159	0.7564	0.9993	1.0000	1.0000
L1/L5	0.2500	0.8382	0.9999	1.0000	1.0000
L2/L5	0.1554	0.9227	1.0000	1.0000	1.0000
L1/L2/L5	0.9850	1.0000	1.0000	1.0000	1.0000

The phase-only dual-epoch success-rate for different frequencies with 1 second time interval and  $\sigma_{\phi_i} = 1$  mm

## Measurement precision

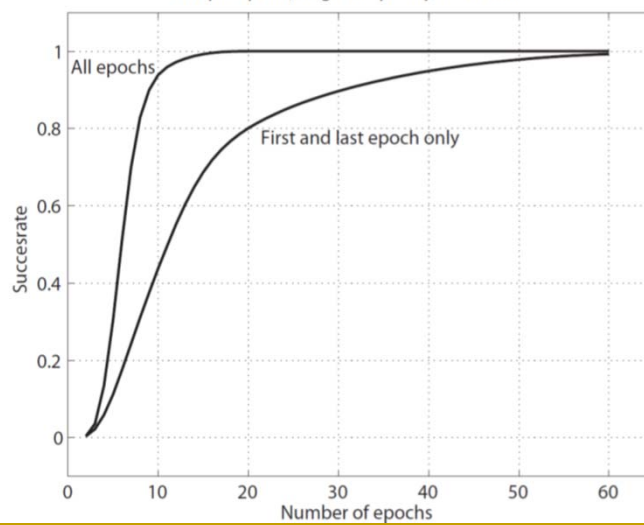


$\sigma_{\phi_i}$ [mm]	Success-rate with 6 satellites
1	0.9993
2	0.6976
3	0.1878

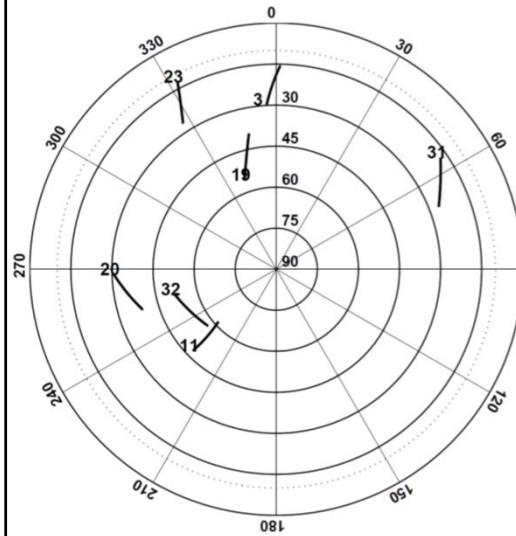
$\sigma_{\phi_i}$ [mm]	Success-rate with 7 satellites
1	1.0000
2	0.9649
3	0.5940

## Phase-only Single Frequency GPS

Multiple epoch, single frequency success rates



## Phase-only experimental results



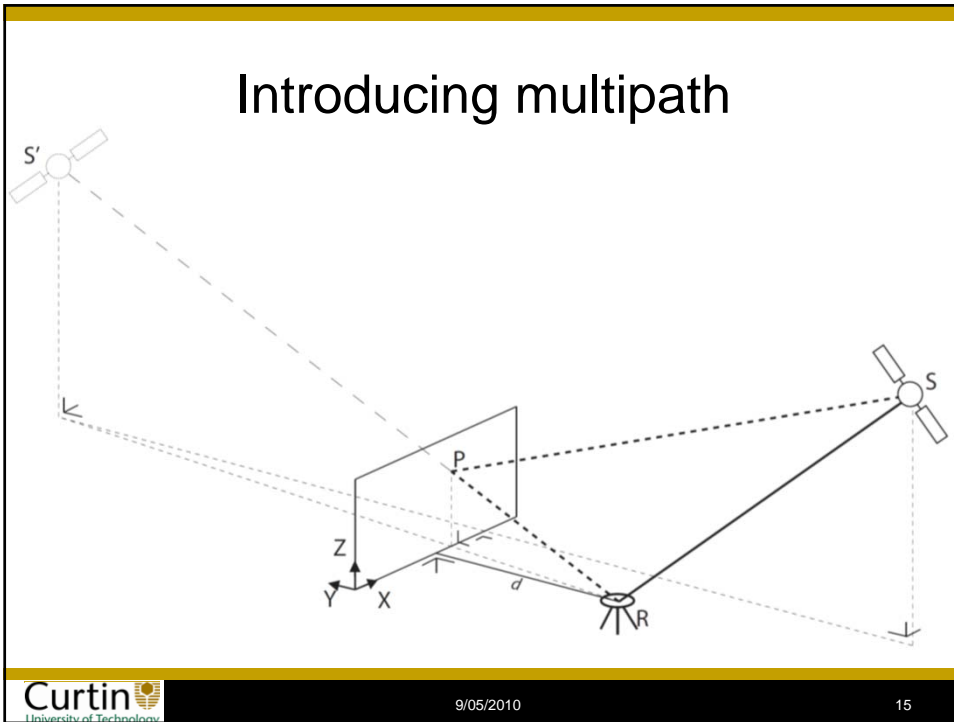
- 7 satellites
- Elevation dependent weighting
- Data rate 1Hz
- GPS L1/L2 code and phase- observations

$$\sigma_{\phi}^s = 0.001 * (1 + 10 \exp \left\{ -\frac{\delta^s}{10^\circ} \right\})$$

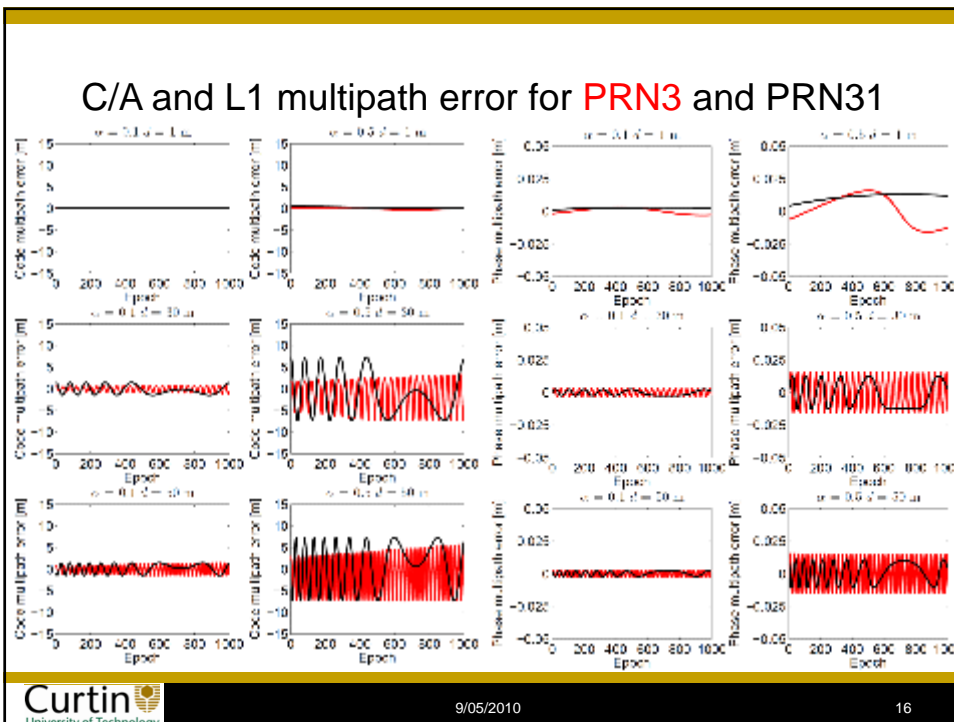
## Robustness - Introducing multipath

- Introduce vertical reflecting surface
- Vary
  - distances to the receiver of the vertical reflecting surface
  - attenuation of the reflected signal with respect to the direct signal

# Introducing multipath



## C/A and L1 multipath error for PRN3 and PRN31





## Empirical phase-only success-rates

$\alpha$	$d[m]$	<i>observables</i>	<i>observables</i>
		<i>L1/L2</i>	<i>C\A/L1/P2/L2</i>
0.1	1	0.998	0.885
	30	0.998	0.552
	50	0.998	0.524
0.5	1	0.683	0.489
	30	0.811	0.552
	50	0.846	0.528

## Ratio of correctly fixed coordinates provided the ambiguities were fixed correctly

$\alpha$	$d[m]$	<i>observables</i>	<i>observables</i>
		<i>L1/L2</i>	<i>C\A/L1/P2/L2</i>
0.1	1	0.998 1.000	0.885 0.000
	30	0.998 1.000	0.552 0.103
	50	0.998 1.000	0.524 0.095
0.5	1	0.683 0.991	0.489 0.000
	30	0.811 0.995	0.552 0.111
	50	0.846 0.998	0.528 0.129

Empirical success rate (1000 samples)

$\frac{\text{Number of epochs with correct baseline coordinates}}{\text{Number of epochs with correct ambiguities}}$

## Ratio of correctly fixed coordinates provided the ambiguities were fixed correctly

$\alpha$	$d[m]$	observables L1/L2	observables C\A/L1/P2/L2
0.1	1	1.000	0.000
	30	1.000	0.103
	50	1.000	0.095
0.5	1	0.991	0.000
	30	0.995	0.111
	50	0.998	0.129

## Integer Least Squares strategy

1. Standard LS adjustment gives real-valued float estimates for unknown (integer) ambiguity and baseline parameters

$$(i) \quad \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}, \quad \begin{bmatrix} Q_{\hat{a}\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}\hat{b}} \end{bmatrix}$$

2. Resolve integer LS estimate for ambiguity parameters

$$(ii) \quad \tilde{a} = \arg \min_{a \in \mathbb{Z}^n} \| \hat{a} - a \|_{Q_{\hat{a}\hat{a}}}^2$$

3. Correct float baseline parameter to obtain fixed baseline solution

$$(iii) \quad \check{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}\hat{a}}^{-1} (\hat{a} - \tilde{a})$$

## Integer Least Squares strategy

1. Standard LS adjustment gives real-valued float estimates for unknown (integer) ambiguity and baseline parameters

$$(i) \quad \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}, \quad \begin{bmatrix} Q_{\hat{a}\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}\hat{b}} \end{bmatrix}$$

2. Resolve integer LS estimate for ambiguity parameters

$$(ii) \quad \tilde{a} = \arg \min_{a \in Z^n} \|\hat{a} - a\|_{Q_{\hat{a}\hat{a}}}^2$$

3. Correct float baseline parameter to obtain fixed baseline solution

$$(iii) \quad \tilde{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}\hat{a}}^{-1} (\hat{a} - \tilde{a})$$

## Ratio of correctly fixed coordinates provided the ambiguities were fixed correctly

$\alpha$	$d[m]$	observables L1/L2	observables C\A/L1/P2/L2
0.1	1	1.000	0.000
	30	1.000	0.103
	50	1.000	0.095
0.5	1	0.991	0.000
	30	0.995	0.111
	50	0.998	0.129

$$\tilde{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}\hat{a}}^{-1} (\hat{a} - \tilde{a})$$

## Conclusions

- Epoch-by-epoch phase-only RTK possible with dual and triple frequency GNSS when respectively 6 and 5 satellites are tracked
- The phase-only RTK model is more robust against multipath than the code and phase RTK model

## On the Robustness of Next Generation Phase-only Real-Time Kinematic Positioning

Lennard Huisman  
Peter J.G. Teunissen  
Dennis Odijk

Curtin University of Technology  
GNSS Research Centre

