# Proper choice of data used for the estimation of datum transformation parameters 

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## Definition of the problem

A complete datum transformation between terrestrial coordinate systems are established by 7-parameter similarity transformation model.
In practice, to minimize the residuals of coincidence between common points, alternative methods such as affine and polynomials can also be applied in two or three dimensional space.

## Definition of the problem

However, the common point coordinates are burdened with distortions due to some random and systematic errors and environmental effects such as crustal movements.
The distortions are absorbed in the transformation parameters estimated. The more the parameters are used for the transformation the more the distortions are absorbed.

## Definition of the problem

To obtain a reliable set of the parameters requires a proper choice of the common points.
What number of the common points should be used? Or,
is their distribution suitable?
These questions are too hard to decide by rule of thumb.
Sometimes, some components of some common points may need to be eliminated while some may be necessary for a reliable solution.

## Method

In this study, we suggests using redundancy numbers for the proper data choice.

## Method

Redundancy number denotes the contribution of data to the degree of freedom of a least square adjustment, and are computed by

$$
r_{i}=\left(\mathbf{I}-\mathbf{A}\left(\mathbf{A}^{\mathrm{T}} \mathbf{P} \mathbf{A}\right)^{-1} \mathbf{A}^{\mathrm{T}} \mathbf{P}\right)_{i i}
$$

```
I = unit matrix;
A = coeffecient matrix of the unknowns;
P = weight matrix of observations
```

Using the redundancy numbers, one can estimate possible distortions

$$
\nabla_{\mathrm{i}}=\frac{\nabla \mathbf{v}_{\mathrm{i}}}{\mathrm{r}_{\mathrm{i}}} \approx \frac{\mathbf{v}_{\mathrm{i}}}{\mathrm{r}_{\mathrm{i}}} \quad \nabla \mathbf{v}_{\mathrm{i}}=\text { distortion impact in the residual } v_{i}
$$

and the absorbed amount of distortions

$$
\mathrm{A}_{\mathrm{i}}=\frac{1-\mathrm{r}_{\mathrm{i}}}{\mathrm{r}_{\mathrm{i}}} \mathrm{v}_{\mathrm{i}}
$$

in each coordinate component of a common point.

## Case Study

20 control points have been determined in Istanbul Metropolitan Area, Turkey. These points are coordinated both in ED50 datum and ITRF94.


## Redundancy numbers of the common points



8 points chosen with respect to residual number $>0.50$


## Redundancy numbers for the selected points



## Both approaches produce equivalent transformation parameters!

| Parameter | Using all the <br> points | Using the <br> proper points |
| :---: | ---: | ---: |
| $\mathbf{t}_{\mathbf{x}}(\mathrm{m})$ | $87.359 \pm 0.017$ | $87.390 \pm 0.027$ |
| $\mathbf{t}_{\mathbf{y}}(\mathrm{m})$ | $91.181 \pm 0.017$ | $91.224 \pm 0.027$ |
| $\mathbf{t}_{\mathbf{z}}$ (meter) | $127.494 \pm 0.017$ | $127.541 \pm 0.027$ |
| $\boldsymbol{\varepsilon}_{\mathbf{x}}$ (arcsecond) | $-3.808 \pm 0.212$ | $-3.658 \pm 0.212$ |
| $\boldsymbol{\varepsilon}_{\mathbf{y}}$ (arcsecond) | $0.132 \pm 0.325$ | $-0.102 \pm 0.380$ |
| $\boldsymbol{\varepsilon}_{\mathbf{z}}$ (arcsecond) | $1.550 \pm 0.190$ | $1.666 \pm 0.220$ |
| $\mathbf{k}$ (ppm) | $3.245 \pm 0.827$ | $3.170 \pm 0.952$ |

Residuals, possible distortion and absorption in coordinates of the selected points


oint No

- residual a pos.dist. a absorption


## Conclusions

Applying this method, one can
$>$ avoid unreliable transformation parameters caused by improper number and distribution of the common points,
$>$ avoid producing redundant common points, and thus provides cost-effective solutions,
$>$ provide further information, such as possible distortion and absorption, about the common points, so one can carry out a more rigorous analysis for the point selection. For instance, some components of some common points may need to be eliminated while some may be kept for a reliable solution.

# Thank you very much for your interest! 

