# Determination of Orthometric Heights with Real Time Kinematic Surveying, Konya Sample 

Ismail SANLIOĞLU, Süleyman Sırrı MARAS and Fatih UYSAL, Turkey

Key words: GPS, Positioning, RTK, geoid determination, surface fitting


#### Abstract

SUMMARY GPS provides measures of heights above a reference ellipsoid, such as the WGS84 ellipsoid, as a perpendicular distance from the ellipsoid surface. However, orthometric heights are measured by perpendicular distance from the geoid. The ellipsoidal heights obtained by GPS do not be used in practical geodesy. To obtain orthometric heights from ellipsoidal heights, it is required that geoid undulations are known. In a mission area, if height differences of geoid rather than geoid heights and orthometric height of one point are known, it is possible to calculate orthometric heights of other points.

In this study, usage of geoid undulations at real time kinematic surveying has been investigated. GPS network had been established according to Big Scale Map and Map Information Production Regulations in Konya metropolitan city at 2008. There are 6 Turkish National Fundamental GPS Network (TUTGA) points, 8 stations graded by C1, 24 stations graded by C2 and 1218 stations graded by C3 appropriately to the Big Scale Map and Map Information Production Regulations (BSH\&MIPR) in the study area, Total area is approximately 1800 square km . Geometric leveling had been carried out to determine orthometric heights of unknown heights of stations using by digital levels.


Konya geoid model had been determined using second order-nine parameter multi-quadratic surface fitting benefiting 1175 control points. Root mean square of model is $\pm 4.5 \mathrm{~cm}$. Geoid undulations of 30 ' $\times 30^{\prime \prime}$ spacing grid points were computed from Konya geoid parameters to use in regional geoid model file (rgm format) for Topcon RTK GPS receivers. The orthometric heights obtained by Topcon receivers have been compared with original heights in the chosen test area.

# Determination of Orthometric Heights with Real Time Kinematic Surveying, Konya Sample 

Ismail SANLIOĞLU, Süleyman Sırrı MARAS and Fatih UYSAL, Turkey

## 1. INTRODUCTION

However, today, the position and the height of a point on the ground can be precisely determined by GPS in WGS84 datum. The spatial information about the point cannot be used directly in the practical engineering and mapping projects. Because the coordinate systems, which are currently in use by several groups of countries, are not referred to the same reference datum and the heights must also be transferred to the geopotential surfaces at the points. The calculations for the point positioning are done on the International Hayford ellipsoid in Turkey and then the coordinates of the point are determined in Gauss-Krueger coordinate system. The height of a point in Turkey is referred to the tide gauge station in Antalya. Therefore, it is presently compulsory to transform the ellipsoidal heights and the positions by GPS to the local datum in use for Turkey. Under the scope of this project, it was studied the usage of geoid undulations at real time kinematic (RTK) surveying. Geoid undulations at the points of interest were determined with respect to RTK and using regional geoid model (rgm) file derived from multi-quadratic interpolation technique and known orthometric heights and RTK results from study were then compared with each other.

## 2. RELATIONSHIP BETWEEN ELLIPSOIDAL AND ORTHOMETRIC HEIGHTS

For geophysical applications, gravity is distinguished from gravitation. Gravity is defined as the resultant of gravitation and the centrifugal force caused by the Earth's rotation. The global mean sea surface is close to one of the equipotential surfaces of the geopotential of gravity. This equipotential surface, or surface of constant geopotential, is called the geoid.

The geoid is that level surface of the Earth's gravity field coincides with the mean ocean surface (Torge, 1980).It is that equipotential surface (surface of fixed potential value) which coincides on average with mean sea level

The geoid is essentially the shape of the earth abstracted from its topographic features. It is an idealized equilibrium surface of sea water, the mean sea level surface in the absence of currents, air pressure variations etc. and continued under the continental masses. The geoid, unlike the ellipsoid, is too complicated to serve as the computational surface on which to solve geometrical problems like point positioning.

The geoid surface is more irregular than the ellipsoid of revolution often used to approximate the shape of the physical Earth, but considerably smoother than Earth's physical surface. The geometrical separation between it and the reference ellipsoid (as GRS-80) is called the geoid undulation. It varies globally between $\pm 110 \mathrm{~m}$. The orthometric height is the distance H along a line of force from a given point A at the physical surface of an object to the geoid.

[^0]2/11

Determination of Orthometric Heights with Real Time Kinematic Surveying, Konya Sample


Figure 1. Relative heighting and relationships among height systems.
Ellipsoidal height of a point on physical earth as P is equal to the distance between the point P and the point where ellipsoid is touched by the line that passes from the point P and is perpendicular to the ellipsoid. The heights obtained by GPS are ellipsoidal heights and does not be used in practical geodesy. This ellipsoid is WGS-84 ellipsoid, semi major axis $a=6378137 \mathrm{~m}$, flattening $\mathrm{a}=1 / 298.257223563$ (Leick, 1995). If reference coordinate system is defined by ITRF datum, the ellipsoidal height is determined on GRS-80 ellipsoid.

### 2.1 Determination of The Geoid Undulations by GPS/Leveling

The ellipsoidal heights are obtained after the process of GPS observations as the orthometric heights after leveling. As can be seen from (Fig. 1), the geoid undulation is calculated by Equation (1) [Liddle, 1989;Ollikainen, 1997].

$$
\begin{equation*}
\boldsymbol{N}=\boldsymbol{h}-\boldsymbol{H} \tag{1}
\end{equation*}
$$

where
$h$ is the ellipsoidal height, and H is the orthometric height.

To obtain orthometric heights from ellipsoidal heights determined by GPS, it is required that geoid undulations $(\mathrm{N})$ are known. In a mission area, if height differences of geoid rather than geoid heights and orthometric height of one point are known, it is possible to calculate orthometric heights of other points. Ellipsoidal heights hA and hB of points A and B can also be determined by means of Equation (2), as below;
Using equation (2)
$\Delta h_{B A}=h_{A_{-}} h_{B}=\left(H_{A-} H_{B}\right)+\left(N_{A_{-}} N_{B}\right)=\Delta H_{B A}+\Delta N_{B A}$

[^1]Determination of Orthometric Heights with Real Time Kinematic Surveying, Konya Sample
and
$\Delta H_{B A}=\Delta h_{B A}-\Delta N_{B A}$
can easily be derived (Sideris, 1990). As ellipsoidal height difference is known by GPS, only one problem is computing of dN geoid undulation difference. The basic geoid undulation determination techniques are

1. Astro-geodetic method.
2. Geopotential models
3. Use of surface gravity in techniques such as Stokes' Integral
4. Geometric or interpolation methods: a local representation of the geoid is obtained only at points which have both leveled (orthometric) heights and heights derived using GPS (ellipsoidal). Geoid heights at other points are found by interpolation.

## 3. MULTIQUADRATIC INTERPOLATION TECHNIQUE

There are several interpolation techniques, which are developed for the transformation of GPS ellipsoidal heights to orthometric heights. Selection of the interpolation technique depends on GPS/Leveling and the number and the position of the points with known height. Here is only explained the multiquadratic interpolation technique, since this technique was chosen as the interpolation technique, which bets fits to the data obtained in this project.

The multiquadratic interpolation technique was suggested by Hardy [1971]. The purpose of this interpolation technique is to define the best matching surface to the region of interest by a single function using the entire control points known. It is also an analytic solution technique. For the application of the technique, a trend surface must be drawn. As a trend surface in this technique, it is appropriate to use first and second degree polynomials [Leberl, 1973].

$$
\begin{equation*}
N_{\text {trend }}(x, y)=a_{0}+a_{1} y+a_{2} x+a_{3} x y+a_{4} y^{2}+a_{5} x y^{2}+a_{6} x^{2} a_{7} x^{2} y+a_{8} x^{2} y^{2} \tag{3}
\end{equation*}
$$

If a second degree polynomial which is not orthogonal, which is used the surface function can be written as Equation (3). Nine control points are required for the solution of this function. If the number of control points is more than $9, a_{i}$ coefficients are calculated using the least square adjustment technique [Inal, 1996]. Taking the advantage of trend surface, the geoid undulation of a point with $x$ and $y$ plane coordinates can be calculated by Equation (4).

$$
\begin{equation*}
N(x, y)=N_{\text {trend }}+\sum_{i=1}^{n} C_{i}\left[\left(x_{i}-x\right)^{2}+\left(y_{i}-y\right)^{2}\right]^{1 / 2} \tag{4}
\end{equation*}
$$

where
$\boldsymbol{n}$ stands for the items control points, and
$\boldsymbol{c}_{\boldsymbol{i}}$ are the coefficients calculated by means of known $\mathrm{N}(\mathrm{x}, \mathrm{y})$ values of the control points

[^2]This interpolation technique does not show any fitting residuals the positions the control points (Çorumluoğlu and et al, 2002).

## 4. REAL TIME KINEMATIC AND KONYA SINGLE REFERENCE STATION FOR RTK

Real Time Kinematic (RTK) satellite navigation is a technique used in land survey and in hydrographic survey based on the use of carrier phase measurements of the GPS, GLONASS and/or Galileo signals. RTK systems use a single base station receiver and a number of mobile units. The base station re-broadcasts the phase of the carrier that it measured, and the mobile units compare their own phase measurements with the ones received from the base station. This allows the units to calculate their relative position to millimeters, although their absolute position is accurate only to the same accuracy as the position of the base station. The typical nominal accuracy for these dual-frequency systems is 1 centimeter $\pm 2$ parts-permillion ( ppm ) horizontally and 2 centimeters $\pm 2 \mathrm{ppm}$ vertically. Also these values may change according capabilities of GPS receiver, communication, distance between base receiver and the rover receiver, distance-dependent biases, namely orbit bias, ionosphere bias and troposphere bias.

In October 2008 Konya Municipality started operating single reference station for RTK positioning, whereby users can receive the corrections throughout the Konya Municipality boundary. Topcon NET G3 reference station receiver was employed. This reference station is controlled by a central server. TopNET 7.12 Reference Station software is used for data processing.

The reference station sends its GPS measurements online to the control room server by dedicated telephone lines. One PC organizes process and archive the data. Communication with the rover is carried out NTRIP protocol, whereby the rover sends its approximate position, calculated in a single positioning mode, to the control station via a GSM mobile call in a National Marine Electronics Association (NMEA) format. Then, the corrections are calculated at the user location and are sent in Radio Technical Commission for Maritime services (RTCM 2.3) format, and used to correct the data to reach cm -level positioning. The coordinates of the reference station is related to the ITRF96 system (reference epoch 1998.00) and the computed rover position can be represented either in this system or in the local one.

## 5. THE MEASUREMENTS IN THE WORKING AREA AND THE EVALUATION

In this study, usage of geoid undulations at real time kinematic surveying has been investigated. GPS network for Konya Photogrammetrical Mapping Project had been established according to Big Scale Map and Map Information Production Regulations in Konya metropolitan city in autumn 2007. There are 6 items Turkish National Fundamental GPS Network (TUTGA) points, 8 stations graded by C1, 24 stations graded by C2 and 1218 stations graded by C3 appropriately to the Big Scale Map and Map Information Production Regulations (BSH\&MIPR) in the study area, Total area is approximately 1800 square km .

GPS observations were collected within the epochs of 10 " interval for 30 minutes observation period at total 1369 points in the working area, with respect to the static GPS observation technique. The coordinates of 151 points were used for fixed adjustment at different sessions. During the collection of GPS observations, 4 items Leica System 1200 receivers and LEIATX1230 antenna and 4 Javad GPS receivers and JPSODYSSEY_I integrated antennas were used. The measurements were completed in 25 days in May and in June 2007. GPS data was processed and adjusted session by session in Pinnacle 1.0 GPS commercial software (Table 1). The antenna library of Pinnacle was used for phase center variation.

Geometric leveling had been carried out to determine orthometric heights of stations by three measurement teams using 3 items Topcon DL-101C, Topcon DL-102C and Sokkisha SDL30 Digital Levels. The measurements completed in 5 months between April-July 2007. The geometric leveling network has 2843 height difference (measurements) and 1885 stations. Thus Konya vertical control network (KVC network) established .Then this network adjusted in one go using 6 points whose coordinates were known in Turkey National Vertical Control Network (TUDKA). The root mean square of leveling adjustment is $\pm 7.56 \mathrm{~mm}$.

Table 1: The processing parameters of Pinnacle Software used in Konya Photogrammetrical Mapping Project

| Elevation Mask (degrees) | 15 |
| :--- | :--- |
| Tropospheric model | Goad\&Goodman |
| Solution type | Auto (Double Difference, L1\&L, |
|  | Iono Free, Wide Lane) |
| Ephemeris | Broadcast |
| Data used | Use Code and Phase |
| Min Measurements (\%) | 10 |
| Max İteration | 6 |
| Measurement Tolerance | 3 |
| Convergence (meters) | 0.001 |
| Contrast Threshold | 0.95 |

The second order-nine parameter multi-quadratic surface was fitted to the topography by using well distributed 1175 control points. Root mean square of surface is $\pm 4.5 \mathrm{~cm}$. The multi-quadratic surface equation is;

$$
\begin{equation*}
N=a_{1}+a_{2} y+a_{3} x+a_{4} y^{2}+a_{5} x y^{2}+a_{6} x y+a_{7} x^{2}+a_{8} x^{2} y+a_{9} x^{2} y^{2} \tag{8}
\end{equation*}
$$

and coefficients of equations are shown Table 2.

[^3]Determination of Orthometric Heights with Real Time Kinematic Surveying, Konya Sample

Table 2. Coefficients of multi-quadratic surface equations

| $\mathrm{a}_{1}=$ | 35.848719452232800 |
| :--- | ---: |
| $\mathrm{a}_{2}=$ | -0.000006229550258 |
| $\mathrm{a}_{3}=$ | 0.000005929540559 |
| $\mathrm{a}_{4}=$ | 0.000000000004456 |
| $\mathrm{a}_{5}=$ | -0.000000000000024 |
| $\mathrm{a}_{6}=$ | -0.000000000230247 |
| $\mathrm{a}_{7}=$ | -0.000000000108540 |
| $\mathrm{a}_{8}=$ | 0.000000000000025 |
| $\mathrm{a}_{9}=$ | 0.00000000000000 |



Figure 2: Entering geoid heights

Geoid undulations of 30 ' ' $\times 30^{\prime \prime}$ spacing grid points were computed from coefficients of multiquadratic surface to use in regional geoid model file (*.rgm) for Topcon RTK GPS receivers. Regional geoid model format is shown in Table 3 and Figure 2. The *.rgm file was converted to a Topcon geoid file (*.gff) by using Topcon link software.

Table 3. Regional Geoid Model Format

| Format Fields | Example |
| :--- | :--- |
| LAT,LON, n_row, n_column, step_lat,step_lon, <br> geoid_direction, ellipsoid; | 37 43 30, 32 21 00, 45, 50, 00 30, 00 30, <br> NE, GRS80; |
| H1 H2 H3 | 35.80735 .81535 .823 |
| H4 H5 H6 | 35.78535 .79535 .805 |
| H7 H8 H9 | 35.76435 .77635 .788 |

### 5.1 Investigating the Accuracy of the Ellipsoidal and Orthometric Height Values

To assess accuracy of orthometric height determination using rover receivers with single GPS reference station and GPS ellipsoidal height and the interpolated orthometric height has to be separately examined, and then jointly evaluated. This helps in showing their performance as well as understating the contribution of each in the total error budget.

A comprehensive testing of GPS heights and horizontal coordinates' determination using the reference station has been carried out by two items Topcon GR3 double frequency receiver on March 16-18 2009. The observation time for RTK is 9 second. The orthometric heights of 34 points obtained by Topcon receivers have been compared with original heights in the chosen test area. But 29 points were used for the orthometric height testing. Because the root mean square of height discrepancies was bigger than $3 \sigma$ in 5 points. However 28 points were used in the comparison of ellipsoidal heights because of same convention (Figure 3).

[^4]Determination of Orthometric Heights with Real Time Kinematic Surveying, Konya Sample

## (KONYA)



Figure 3. Topograhy of Konya City, created by GMT (Wessel and Smith, 1998).
Comparing the point coordinates of 29 points which were previously precisely determined with their coordinates determined by RTK gave horizontal positioning accuracy 1.8 cm in easting, 2 cm in northing. These surveys also prove to be consistent in terms of the internal precision of their final output results (i.e. statistically compatible).

The height difference between the highest and lowest points in the test area was approximately 377.4 meters. Figure 4 shows the difference in orthometric height between estimated leveling adjustment and obtained RTK with geoid model (the geoid model + GPS

Ismail Sanlioğlu, Süleyman Sırrı Maras and Fatih Uysal
Determination of Orthometric Heights with Real Time Kinematic Surveying, Konya Sample
ellipsoidal heights). The height discrepancies were in the range $\pm 18 \mathrm{~cm}$. The statistics of these discrepancies are given in Table 4 where the mean value was 0.031 m with random distribution of the differences, and no systematic errors were observed. The average value of the absolute differences was $\pm 1.9 \mathrm{~cm}$ with $\pm 2.33 \mathrm{~cm} \mathrm{rms}$ of differences


Fig. 4 Height differences between RTK using regional geoid model and estimated leveling adjustment (m)

Table 4. Height differences between RTK using regional geoid model and known heights (m)

|  | Average | Average of <br> Absolute values | Maximum <br> Discrepancy | Minimum <br> Discrepancy | RMS <br> $\boldsymbol{\sigma}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Orthometric | 0.031 m | 0.094 m | 0.184 m | 0.010 m | 0.107 m |
| Ellipsoidal | 0.008 m | 0.039 m | 0.101 m | 0.002 m | 0.052 m |

The results show that there are no systematic errors either in ellipsoidal height determination by GPS. The difference in orthometric height between estimated leveling adjustment and obtained RTK with geoid model (the geoid model + GPS ellipsoidal heights) can be attributed to
a) random noise in the values of $\mathrm{h}, \mathrm{H}, \mathrm{N}$ in KVC network
b) datum inconsistencies and other possible systematic distortions in the three height data sets (e.g. long-wavelength systematic errors in N , distortions in the vertical datum due to an over constrained adjustment of the leveling network, deviation between gravimetric geoid and reference surface of the leveling datum) (El-Mowafy et al, 2006).

## 6. CONCLUSIONS

The feasibility of determining real-time orthometric heights by RTK at the cm-level accuracy level is shown in Konya. The RTK GPS survey technique may be an efficient and accurate alternative to the leveling techniques currently being used. On average the results agree to within 10 cm . The regional geoid model was estimated from second degree multi-quadratic

[^5]9/11
Determination of Orthometric Heights with Real Time Kinematic Surveying, Konya Sample
surface in this study. According to ellipsoidal heights, the deviations of orthometric heights are double big. The estimated surface may be incompatible with topography. However, further tests on the larger network with a greater number of stations may provide more insight to RTK with geoid model method as an alternative to leveling.

## ACKNOWLEDGEMENTS

The Map Department of Konya Municipality is acknowledged for the dedicated help and efforts in completing this study. Survey Engineer and Chief of Map Department Celal Can, Barbaros Ipek, and their team are gratefully acknowledged for their help in the land surveying, and data collection. The Manager of OSTEM Mapping Corporation and Eng. Erhan Civcik also thanked receiving data as regards GPS network for Konya Photogrammetrical Mapping Project.

## REFERENCES

Çorumluoglu, Ö., Inal C., Ceylan, A., Sanlıoğlu, I., Kalayci, I., 2002, Determination of Geoid Undulations in the Region of Konya, International Symposium on GIS, September 23-26, Istanbul, Turkey
El-Mowafy, A., Fashir, H., Al Habbai, A. Al Marzooqi, Y. and Babiker, T. 2006, RealTime Determination of Orthometric Heights Accurate to the Centimeter Level using a Single GPS Receiver: Case Study, Journal of Surveying Engineering, Vol. 132, No. 1, pp. 1-6.
Hardy, R.L., 1971, Multiquadratic Equation of Topography and Other Irregular Surfaces, Journal of Geophysical Research, Vol.76, No 8
Inal, C., 1996, Height Determination from GPS using local geoid, Journal of Engineering and Architectural Faculty of S.U., Vol 11, No 2, pp 15-21
Leberl, F., 1973, Interpolation in a Square Grid DTM, ITC Journal, 1973-75
Leick, A., 1995, GPS Satellite Surveying, Second Edition, John Wiley\&Sons, Inc.
Liddle, L. A., 1989, Orthometric Height Determination by GPS, Surveying and Mapping, Vol 49, No 1,
Ollikanen, M., 1997, Determination of Orthometric Heights using GPS leveling, Publication of the Finnish Geodetic Institute, No 23, Kirkkonum
Sideris, M. G., 1990. The role of the geoid in one-, two-, and three-dimensional network adjustments. CISM Journal ACSGC, 44 (1) : 9-18.
Torge, W., 1980. Geodesy, Walter de Gruyter, Berlin-New York.
Wessel, P. and Smith.,W. H. F., 1998, New, improved version of the Generic Mapping Tools Released, EOS Trans. AGU, 79, 579.

## CONTACTS

Assist. Prof. Dr. Ismail Sanhoğlu
Selcuk University, Engineering and Arhitecture Faculty
Geodesy and Photogrammetry Department
University Lecturer, Süleyman Sırrı Maras
Selcuk University, Technical Sciences Vocational School of Higher Education
Mapping-Cadastre Programme, Konya
Research Assist. Fatih Uysal
Selcuk University, Engineering and Arhitecture Faculty
Geodesy and Photogrammetry Department
Konya
TURKEY
Tel. +90 3322231940
Fax +903322410635
Email: sanlioglu@selcuk.edu.tr, fuysal@selcuk.edu.tr, ssm@selcuk.edu.tr,

[^6]
[^0]:    TS3 - Geoid Modelling
    Ismail Sanlioğlu, Süleyman Sırrı Maras and Fatih Uysal

[^1]:    TS3 - Geoid Modelling
    Ismail Sanlioğlu, Süleyman Sırrı Maras and Fatih Uysal

    FIG Working Week 2009
    Surveyors Key Role in Accelerated Development
    Eilat, Israel, 3-8 May 2009

[^2]:    TS3 - Geoid Modelling

[^3]:    TS3 - Geoid Modelling
    Ismail Sanlioğlu, Süleyman Sırrı Maras and Fatih Uysal

    FIG Working Week 2009
    Surveyors Key Role in Accelerated Development
    Eilat, Israel, 3-8 May 2009

[^4]:    TS3 - Geoid Modelling
    Ismail Sanlioğlu, Süleyman Sırrı Maras and Fatih Uysal

    FIG Working Week 2009
    Surveyors Key Role in Accelerated Development
    Eilat, Israel, 3-8 May 2009

[^5]:    TS3 - Geoid Modelling
    Ismail Sanlioğlu, Süleyman Sırrı Maras and Fatih Uysal

    FIG Working Week 2009
    Surveyors Key Role in Accelerated Development
    Eilat, Israel, 3-8 May 2009

[^6]:    TS3 - Geoid Modelling
    Ismail Sanlioğlu, Süleyman Sırrı Maras and Fatih Uysal
    Determination of Orthometric Heights with Real Time Kinematic Surveying, Konya Sample
    FIG Working Week 2009
    Surveyors Key Role in Accelerated Development
    Eilat, Israel, 3-8 May 2009

