Geodetic Deformation Analysis with Respect to Observation Imprecision

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Key words: Observation imprecision, systematic errors, hypothesis testing, fuzzy intervals, uncertainties, deformation analysis

SUMMARY

The significance of estimated point and object movements in geodetic deformation analysis depends essentially on the quality of the observations and analysis techniques. A comprehensive modeling of the complete analysis process from the original observations to the parameters of interest requires an adequate consideration and propagation of all sources of uncertainty. In this study, the uncertainty budget of the observations is assumed to comprise two independent types of uncertainty: random variability and imprecision. The first one is well known; it is described by random variables. The second one is due to remaining systematic deviations between the observations and the geodetic model; it is modeled using fuzzy intervals. In order to take observation imprecision into account, both types of uncertainty are superposed to consistently extend the standard analysis techniques which are exclusively based on random variables: fuzzy intervals serve now as basic quantities; their midpoints are random variables representing the classical observations.

This study shows for the first time a completely evaluated deformation analysis where both types of uncertainty (random variability and imprecision) are considered in a comprehensive way. We focus on the determination of relevant quantities and parameters and on statistical hypothesis testing in case of imprecision, in order to check the accordance of the collected data with the assumptions met in the model. Further on, a detailed application example illustrates the theoretical concept from a practical point of view, including the comparison to the pure stochastic case.

SUMMARY (German)

Die Aussagekraft von Punktkoordinaten und Objektbewegungen hängt im Wesentlichen von der Qualität der Beobachtungen und Auswertemethoden ab. Insbesondere die Fortpflanzung der Unsicherheiten von den originären Beobachtungen auf die zu schätzenden Parameter bedarf einer adäquaten Berücksichtigung des gesamten Unsicherheitshaushaltes. Aus diesem Grund werden in diesem Beitrag zwei unabhängige Arten von Unsicherheiten berücksichtigt. Zum einen die stochastische Variation der Messwerte und zum anderen verbleibende Restsystematiken zwischen den Modellannahmen und den realen Beobachtungen (Impräzision). Beide Arten von Unsicherheit werden zum ersten Mal in einem übergreifenden Ansatz auf eine geodätische Deformationsanalyse übertragen, wobei der Schwerpunkt der Untersuchungen auf der Parameterschätzung und den Hypothesentests liegt. Abschließend wird ein Beispiel für eine komplette Deformationsanalyse unter Berücksichtigung beider Unsicherheiten aufgezeigt und mit einer rein stochastischen Auswertung kritisch verglichen.

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1. INTRODUCTION

One very important aim of uncertainty modeling in engineering applications is to get reliable informations about the uncertainty budget of estimated parameters. Therefore, different types of uncertainty are considered, which are propagated from the original observations to the parameters of interest. In this study, the uncertainty budget of the observations is assumed to comprise two independent types of uncertainty: random variability and imprecision. The first one is well known; it is described by random variables. The second one is due to remaining systematic deviations between the observations and the geodetic model.

Different types of uncertainty need different laws of uncertainty propagation. Whereas the stochastic part is treated with the law of propagation of covariances, imprecision is propagated with a sensitivity analysis (see Section 3). Both types of uncertainty can be modeled in a comprehensive way, using fuzzy intervals (see Section 2). This procedure is in full accordance with the international recommendations *"Guide to the expression of uncertainty in measurements" (GUM)*, cf. (ISO 1995). Note that the treatment of systematic errors is different as proposed within the GUM.

In this study, the methods to describe the extended uncertainty budget and the influence of remaining systematics (imprecision) during the measurement process are presented and discussed. It is shown that the consideration of imprecision is an additive term of uncertainty and leads to more reluctant rejections of the null hypothesis than in the pure stochastic case.

The presented strategies for the evaluation of a complete deformation analysis can be transferred to many other engineering applications, considering that the parameters must be estimated within a least-squares adjustment, cf. e. g., (Kutterer 2006).

2. UNCERTAINTY MODELING USING FUZZY INTERVALS

Fuzzy theory (Zadeh 1965) and interval mathematics have proven to be an appropriate solution for the description of remaining systematics. Recently, many procedures have been introduced in different engineering applications, cf. e. g., (Kieffer et. al. 2000; Morales and Son 1998) and (Muhanna and Mullen 2001), incl. discussions about combined approaches in fuzzy theory, interval mathematics and probability theory, e. g., (Ferson et al. 2002).

While modeling the uncertainty budget in a comprehensive way, both mentioned uncertainties of the here presented approach are characterized with a special case of fuzzy theory, the so called fuzzy randomness (Bandemer and Näther 1992; Möller and Beer 2004; Viertl 1996). We assume a precise stochastic component what is standard in geodetic data analysis. This component is superposed by imprecision due to unknown remaining systematic errors. Note that imprecise quantities are exclusively modeled in terms of fuzzy intervals.

A fuzzy interval $\tilde{A}$ is uniquely defined by its membership function $m_{\tilde{A}}(x)$ over the set $\mathbb{R}$ of real numbers with a membership degree between 0 and 1:

$$\tilde{A} := \left\{ (x, m_{\tilde{A}}(x)) \middle| x \in \mathbb{R} \right\} \qquad \text{with} \qquad m_{\tilde{A}} : \mathbb{R} \rightarrow [0,1] \tag{1}$$

The membership function of a fuzzy interval can be described by its left (*L*) and right (*R*) reference function (see also Fig. 1)

$$m_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{x_m - x - r}{c_l}\right), & x < x_m - r \\ 1, & x_m - r \le x \le x_m + r \\ R\left(\dfrac{x - x_m - r}{c_r}\right), & x > x_m + r \end{cases} \tag{2}$$

with x_m denoting the midpoint, r its radius, and c_l, c_r the spread parameters of the monotonously decreasing reference functions (convex fuzzy intervals). Fuzzy intervals serve now as basic quantities; their midpoints x_m are considered in the following as random variables representing the classical observations and their spread which describes the range of imprecision.

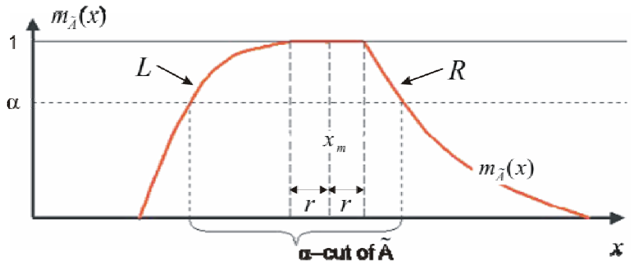

Figure 1: Fuzzy interval and its α-cut

The *α-cut* of a fuzzy interval $\tilde{A}$ is defined by:

$$\tilde{A}_\alpha := \left\{ x \in X \middle| m_{\tilde{A}}(x) \ge \alpha \right\}, \tag{3}$$

with $\alpha \in [0,1]$. Each *α-cut* represents in case of monotonously decreasing reference functions a classical interval. The lower bound $\tilde{A}_{\alpha,\min}$ and the upper bound $\tilde{A}_{\alpha,\max}$ of an *α-cut* are obtained as:

$$\tilde{A}_{\alpha,\min} = min\left(\tilde{A}_\alpha\right), \tag{4}$$

$$\tilde{A}_{\alpha,\max} = max\left(\tilde{A}_\alpha\right). \tag{5}$$

Throughout the paper we assume symmetric fuzzy intervals. Hence, an equal representation of symmetric *α-cuts* can be found by the midpoint A_m and radius $\tilde{A}_{\alpha,r}$ representation:

$$\tilde{A}_{\alpha,\min} = A_m - \tilde{A}_{\alpha,r}\,, \qquad (6)$$

$$\tilde{A}_{\alpha,\max} = A_m + \tilde{A}_{\alpha,r}\,. \qquad (7)$$

The integral over all α*-cuts* equals the membership function:

$$m_{\tilde{A}}(x) = \int_0^1 m_{\tilde{A}_\alpha}(x)\,d\alpha \qquad (8)$$

Furthermore, basic operations on fuzzy intervals are the *intersection* and the *complement*; they are defined through the following membership functions:

Intersection: $\tilde{C} = \tilde{A} \cap \tilde{B} \Leftrightarrow m_{\tilde{A}\cap\tilde{B}}(x) = \min\left(m_{\tilde{A}}(x), m_{\tilde{B}}(x)\right) \quad \forall\ x \in \mathbb{R}$ (9_a)

Complement: $\tilde{C} = \tilde{A}^C \Leftrightarrow m_{\tilde{A}^C}(x) = 1 - m_{\tilde{A}}(x) \quad \forall\ x \in \mathbb{R}$ (9_b)

There are also arithmetic rules which can be directly applied to fuzzy intervals. For further information on fuzzy-theory and interval mathematics, cf. e. g., (Alefeld and Herzberger 1983; Bandemer and Näther 1992; Kaufmann and Gupta 1991).

3. ANALYIS OF THE OBSERVATIONS AND ESTIMATED PARAMETERS

In order to take observation imprecision into account, the influence parameters $\mathbf{p}$ of the preprocessing steps of the raw observations, e. g. temperature, pressure, additive constants etc., are described by fuzzy intervals ($\mathbf{p} \rightarrow \tilde{\mathbf{p}}$), see (Schön 2003) for terrestrial measurements and (Schön and Kutterer 2003, 2005a, 2006) for GPS measurements. The membership functions are based on expert knowledge, on manufacturer’s introductions and on empirical studies. Figure 2 gives an overview for the sequence of work steps in an adjustment with respect to observation imprecision. The basic aspects of the formalism are summarized in the following subsections; they are based on a detailed description in interval mathematical notation (Schön 2003).

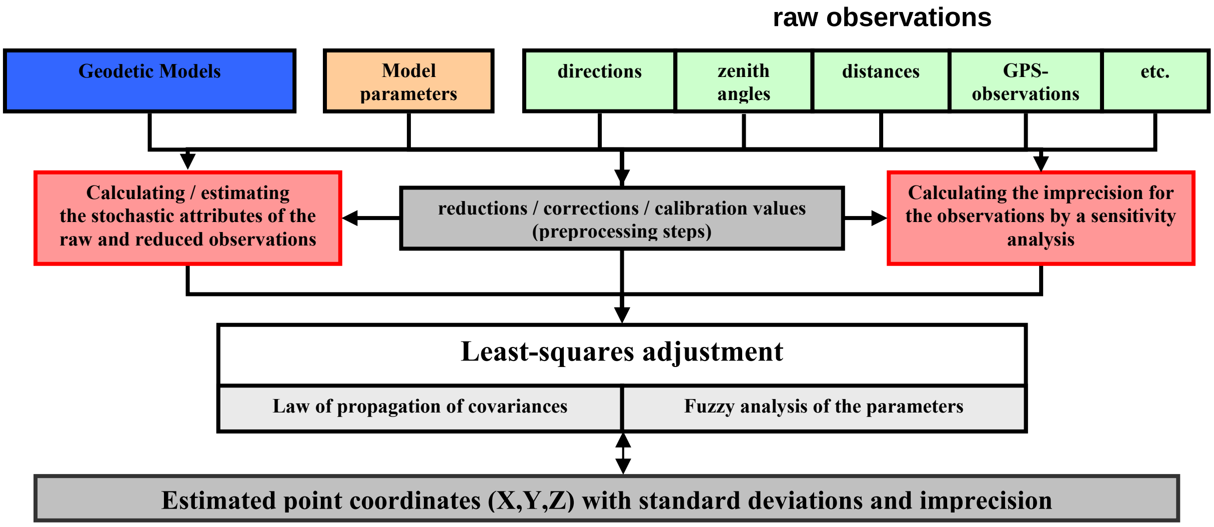

Figure 2: Least-squares adjustment with respect to observation imprecision

3.1 Sensitivity Analysis of the Observations

The effects of the imperfect knowledge of the influence parameters on the observations $\mathbf{l}$ are computed with a sensitivity analysis. This procedure leads to fuzzy intervals for the observations ($\mathbf{l} \rightarrow \tilde{\mathbf{l}}$):

$$\tilde{\mathbf{l}} = \boldsymbol{f}(\tilde{\mathbf{p}}), \tag{10}$$

with the midpoint of the fuzzy intervals representing the pure stochastic case:

$$\mathbf{l}_m = \boldsymbol{f}(\mathbf{p}_m). \tag{11}$$

The propagation of imprecision is based on the *α-cuts* $\tilde{\mathbf{p}}_\alpha$ of the imprecise influence parameters. Therefore, the function between the influence parameters $\tilde{\mathbf{p}}$ and the observations $\tilde{\mathbf{l}}$ are linearized:

$$\tilde{\mathbf{l}}_{\alpha,\min} = f(\mathbf{p}_m) - |\mathbf{F}|\left(\tilde{\mathbf{p}}_{\alpha,r}\right), \tag{12\_a}$$

$$\tilde{\mathbf{l}}_{\alpha,\max} = f(\mathbf{p}_m) + |\mathbf{F}|\left(\tilde{\mathbf{p}}_{\alpha,r}\right), \tag{12\_b}$$

with the matrix of partial derivatives $\mathbf{F} = \dfrac{\partial \mathbf{l}}{\partial \mathbf{p}}$ and $|\square|$ denoting the element by element absolute value of the matrix. The imprecise vector of observation is then obtained as:

$$m_{\tilde{\mathbf{l}}}(x) = \int_0^1 m_{\tilde{\mathbf{l}}_\alpha}(x)\,d\alpha \qquad \text{and} \qquad m_{\tilde{\mathbf{l}}_\alpha} = \left[\tilde{\mathbf{l}}_{\alpha,\min}, \tilde{\mathbf{l}}_{\alpha,\max}\right]. \tag{13}$$

In case of linear reference functions for the imprecise influence parameters, the propagation of imprecision must only be applied for the *α-cuts* $\tilde{\mathbf{p}}_\alpha$ with $\alpha = 0$ and $\alpha = 1$. Otherwise, the imprecise vector of reduced observations is constructed based on a sufficient number of *α-cuts* from Equation (12) and (13).

3.2 Fuzzy evaluation of the parameter vector

In order to transfer the uncertainty from the imprecise influence parameters $\tilde{\mathbf{p}}$ to the parameters of interest $\mathbf{x}$, the least-squares adjustment has to be extended to the case of imprecision. Strictly speaking, the imprecise parameter vector must be computed by means of fuzzy theory (see Equation 15). In order to avoid overestimation, the evaluation of the parameter vector is based on the imprecise influence parameters $\tilde{\mathbf{p}}$. The midpoints of the estimated parameters $\hat{\mathbf{x}}_m$ are obtained by the Gauß-Markov-Model, cf. (Koch 1999):

$$\hat{\mathbf{x}}_m = \boldsymbol{f}(\mathbf{l}_m, \mathbf{x}_0) = \mathbf{x}_0 + (\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}(\mathbf{l}_m - \mathbf{a}_0)\,, \tag{14}$$

with the $n \times u$ column regular design matrix $\mathbf{A}$, the $n \times 1$ vector of approximate values $\mathbf{x}_0$ of the parameters $\mathbf{x}$, the $n \times n$ regular weight matrix $\mathbf{P}$ and the $n \times 1$ vector of approximate observations $\mathbf{a_0}$. Then, the imprecise vector of estimated parameters $\tilde{\hat{\mathbf{x}}}$ is constructed, based on a sufficient number of *α-cuts*:

$$\tilde{\hat{\mathbf{x}}}_{\alpha,\min} = \mathbf{x}_0 + (\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}\left[\mathbf{l}_m - \mathbf{a}_0 - |\mathbf{F}|\left(\tilde{\mathbf{p}}_{\alpha,r}\right)\right], \tag{15\_a}$$

$$\tilde{\hat{\mathbf{x}}}_{\alpha,\max} = \mathbf{x}_0 + (\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}\left[\mathbf{l}_m - \mathbf{a}_0 + |\mathbf{F}|\left(\tilde{\mathbf{p}}_{\alpha,r}\right)\right], \tag{15\_b}$$

$$m_{\tilde{\hat{\mathbf{x}}}}(x) = \int_0^1 m_{\tilde{\hat{\mathbf{x}}}_\alpha}(x)\,d\alpha \qquad \text{and} \qquad m_{\tilde{\hat{\mathbf{x}}}_\alpha} = \left[\tilde{\hat{\mathbf{x}}}_{\alpha,\min}, \tilde{\hat{\mathbf{x}}}_{\alpha,\max}\right]. \tag{15\_c}$$

The presented approach is also transferable to a column-singular design matrix $\mathbf{A}$, by using the pseudo inverse for the normal equations, cf. (Neumann et al. 2006) for the interval mathematical treatment. The parameter vector is exact component by component but overestimates the correct range of values which is a convex polyhedron (zonotope). See (Schön 2003) and (Schön and Kutterer 2005b) for a detailed description on zonotopes.

4. HYPOTHESIS TESTING WITH RESPECT TO OBSERVATION IMPRECISION

In this section, a short introduction to hypothesis testing with respect to observation imprecision is presented. We start with the pure stochastic case, where a quadratic form may be given by the following equation:

$$\mathbf{y}^T\mathbf{\Sigma}_{\mathrm{yy}}^{-1}\mathbf{y} \sim \chi^2(f, \lambda)\,, \tag{16}$$

with the assumed normal distributed vector of reduced observations $\mathbf{y} = \mathbf{l} - \mathbf{a_0}$ and its associated variance covariance matrix Σ_{yy}. Hence, the quadratic form follows a chi-square distribution with $\mathrm{f} = \mathrm{rank}(\mathbf{\Sigma}_{\mathrm{yy}})$ degrees of freedom and the non-centrality parameter λ.

In the following, we pick up the above-mentioned idea to bring all problems back to the imprecise influence parameters in order to avoid overestimation. Therefore, the general form of a linear hypothesis has to be introduced as a function of the influence parameters $\mathbf{p}$. First, the alternative hypothesis H_A is given as:

$$H_A: \quad \mathbf{Bx} = \mathbf{w}, \tag{17}$$

provided that $\mathbf{Bx}$ must be a testable hypothesis, cf. (Koch 1999) for detailled introductions concerning the matrix $\mathbf{B}$. The alternative hypothesis must be compared with the null hypothesis

$$H_0: \quad \mathrm{E}(\mathbf{y}) = \mathbf{Ax}, \tag{18}$$

where the expected value of the reduced observations $\mathrm{E}(\mathbf{y})$ equals $\mathbf{Ax}$. Examples for the Matrix $\mathbf{B}$ in hypothesis testing are given in (Koch 1999) and (Welsch et al. 2000). This leads after a few calculation steps to a quadratic form:

$$T = (\mathbf{B}^\mathrm{T}\hat{\mathbf{x}} - \mathbf{w})^T \left[\mathbf{B}^\mathrm{T}\left(\mathbf{A}^\mathrm{T}\mathbf{PA}\right)^+ \mathbf{B}\right]^+ (\mathbf{B}^\mathrm{T}\hat{\mathbf{x}} - \mathbf{w}) \sim \chi^2(h,0) \text{ under } \mathrm{H}_0, \tag{19}$$

that follows under the null hypothesis a central chi-square distribution ($\lambda = 0$) with $h = \mathrm{rank}\left[\mathbf{B}^\mathrm{T}\left(\mathbf{A}^\mathrm{T}\mathbf{PA}\right)^+ \mathbf{B}\right]$ degrees of freedom. This quadratic form from Equation (19) has to be converted to a quadratic form of imprecise influence parameters $\tilde{\mathbf{p}}$; it is obtained as:

$$\tilde{T}_{\alpha,\min} = \min\left(\begin{bmatrix}\Delta\mathbf{p}\\ \mathbf{y}_m\\ \mathbf{w}\end{bmatrix}^T \begin{bmatrix}\mathbf{F}^\mathrm{T}\mathbf{K}^\mathrm{T}\mathbf{DKF} & -\mathbf{F}^\mathrm{T}\mathbf{K}^\mathrm{T}\mathbf{DK} & -\mathbf{F}^\mathrm{T}\mathbf{K}^\mathrm{T}\mathbf{D}\\ -\mathbf{K}^\mathrm{T}\mathbf{DKF} & \mathbf{K}^\mathrm{T}\mathbf{DK} & -\mathbf{K}^\mathrm{T}\mathbf{D}\\ -\mathbf{DKF} & -\mathbf{DK} & \mathbf{D}\end{bmatrix} \begin{bmatrix}\Delta\mathbf{p}\\ \mathbf{y}_m\\ \mathbf{w}\end{bmatrix}\right), \tag{20\_a}$$

$$\tilde{T}_{\alpha,\max} = \max\left(\begin{bmatrix}\Delta\mathbf{p}\\ \mathbf{y}_m\\ \mathbf{w}\end{bmatrix}^T \begin{bmatrix}\mathbf{F}^\mathrm{T}\mathbf{K}^\mathrm{T}\mathbf{DKF} & -\mathbf{F}^\mathrm{T}\mathbf{K}^\mathrm{T}\mathbf{DK} & -\mathbf{F}^\mathrm{T}\mathbf{K}^\mathrm{T}\mathbf{D}\\ -\mathbf{K}^\mathrm{T}\mathbf{DKF} & \mathbf{K}^\mathrm{T}\mathbf{DK} & -\mathbf{K}^\mathrm{T}\mathbf{D}\\ -\mathbf{DKF} & -\mathbf{DK} & \mathbf{D}\end{bmatrix} \begin{bmatrix}\Delta\mathbf{p}\\ \mathbf{y}_m\\ \mathbf{w}\end{bmatrix}\right), \tag{20\_b}$$

$$m_{\tilde{T}}(x) = \int_0^1 m_{\tilde{T}_\alpha}(x)\,d\alpha \qquad \text{and} \qquad m_{\tilde{T}_\alpha} = \left[\tilde{T}_{\alpha,\min}, \tilde{T}_{\alpha,\max}\right]. \tag{20\_c}$$

with $\Delta\mathbf{p} \in \Delta\tilde{\mathbf{p}}_\alpha = \left[\tilde{\mathbf{p}}_{\alpha,\min} - \mathbf{p}_\mathrm{m}\ ,\ \tilde{\mathbf{p}}_{\alpha,\max} - \mathbf{p}_\mathrm{m}\right]$, $\mathbf{K} = \mathbf{B}^T\left(\mathbf{A}^\mathrm{T}\mathbf{PA}\right)^+ \mathbf{A}^\mathrm{T}\mathbf{P}$, $\mathbf{D} = \left[\mathbf{B}^\mathrm{T}\left(\mathbf{A}^\mathrm{T}\mathbf{PA}\right)^+ \mathbf{B}\right]^+$ and $\mathbf{y}_m$ the midpoint of the reduced observations. The fuzzy evaluation of the quadratic form from Equation (20) is based on Zadeh's extension principle (Zadeh 1965), which can be equivalently replaced by the min-max operator of an optimization algorithm, cf. (Dubois and Prade 1980, p. 37) for the theoretical concept and (Möller and Beer 2004) for applications in civil engineering. The optimization problem can be solved, e. g., with a standard Newton algorithm, cf. (Coleman and Li 1996). Figure 3 shows a constructed test value $\tilde{T}$ and the comparison of the imprecise test value with the imprecise regions of acceptance $\tilde{A}$ and rejection $\tilde{R}$.

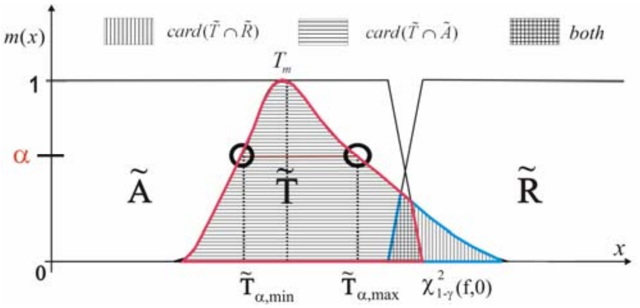

Figure 3: Comparison of the test value $\tilde{T}$ with the regions of acceptance $\tilde{A}$ and rejection $\tilde{R}$

(Neumann et al. 2006) demonstrate the evaluation of the above (see Equation 20) given imprecise quadratic function for outlier and global tests. A congruence test with respect to observation imprecision is computed in (Neumann and Kutterer 2006). Whereas the influence of imprecision on the test decision for a smaller number of observations is unimportant, it gets more important for a larger number of observations. This is in full accordance to the theoretical concept, because the goodness of fit for the stochastic uncertainty of the parameters increases with the number of observations.

4.1 Test decision based on the card criterion

The final test decision is based on the set-theoretical comparison of the imprecise test value (constructed using an α-cut optimization algorithm) with the region of acceptance $\tilde{A}$ and the region of rejection $\tilde{R}$ (see Fig. 3), cf. (Kutterer 2004) and (Neumann et al. 2006) for detailed explanations. The hypotheses are defined by

$$T_m \sim \chi^2(f,\lambda)\ ;\quad \lambda \begin{cases} =0 & \mid \mathrm{H}_0 \text{ the null hypothesis,} \\ \neq 0 & \mid \mathrm{H}_\mathrm{A} \text{ the alternative hypothesis,} \end{cases} \tag{21}$$

with the non-centrality parameter λ. The midpoint of the test value follows under the null hypothesis a central chi-square distribution with f degrees of freedom. The regions of acceptance $\tilde{A}$ and rejection $\tilde{R} = \tilde{A}^C$ are defined as fuzzy intervals. The degree of the rejectability $\rho_R(\tilde{T})$ of the null hypothesis H_0 under the condition of $\tilde{T}$ is computed based on the degree of agreement of the test value with the region of rejection $\gamma_{\tilde{R}}(\tilde{T})$ and the degree of disagreement of the test value with the region of acceptance $\delta_{\tilde{A}}(\tilde{T})$. We use the card criterion, because it allows a more suitable description of the degree of agreement between two fuzzy intervals. This leads to the equations given below (see Fig. 3):

$$\gamma_{\tilde{R}}(\tilde{T}) = \frac{card\left(\tilde{T} \cap \tilde{R}\right)}{card\left(\tilde{T}\right)} \quad \text{and} \quad \delta_{\tilde{A}}(\tilde{T}) = 1 - \frac{card\left(\tilde{T} \cap \tilde{A}\right)}{card\left(\tilde{T}\right)} \tag{22\_a}$$

$$\rho_{\tilde{R}}(\tilde{T}) = \min\left(\gamma_{\tilde{R}}(\tilde{T}), \delta_{\tilde{A}}(\tilde{T})\right) \tag{22\_b}$$

For the final test decision, the degree of rejectability $\rho_{\tilde{R}}(\tilde{T})$ of the null hypothesis has to be compared with a suitable critical value $\rho_{crit} \in [0,1]$:

$$\rho_{\tilde{R}}(\tilde{T}) > \rho_{crit} \Rightarrow \text{reject } H_0 \quad (23)$$

The test is only rejected, if the test value agrees with the region of rejection and disagrees with the region of acceptance. This is in full accordance with the theoretical expectations, where observation imprecision is an additive term of uncertainty during the measurement process. The choice of ρ_{crit} depends on the particular application and must be based on expert knowledge. For outlier detection we propose to choose $\rho_{crit} > 0.5$ and for safety-relevant measures $\rho_{crit} \to 0$.

5. EXAMPLE: DEFORMATION ANALYIS FOR THE MONITORING OF A LOCK

This section shows a completely evaluated deformation analysis with respect to observation imprecision. We focus in the given example on the 2-dimensional geodetic network of the lock "Uelzen I". The network is composed of eight control points around the lock and four points (101-104) on top of the lock (see Figure 4). For further informations about the monitoring network, cf. (Neumann et al. 2006) and (Neumann and Kutterer 2006).

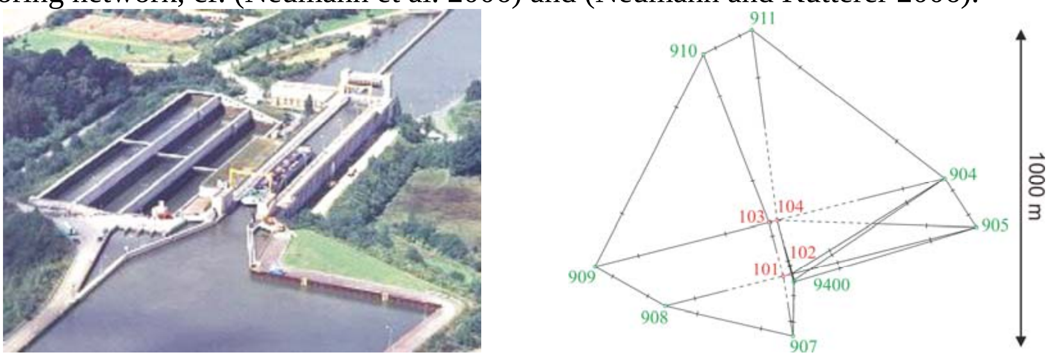

Figure 4: The lock "Uelzen I" and the geodetic monitoring network

Table 1 and 2 show some typical orders of magnitude for the standard deviations σ and interval radii $\tilde{\mathbf{l}}_{\alpha,r}$ (α-level of zero) of the observations and interval radii $\tilde{\hat{\mathbf{x}}}_{\alpha,r}$ (α-level of zero) of the parameters, obtained by the described methods from Sections 3.1 and 3.2.

	Distances	Zenith angles	Horizontal angles
$\tilde{\mathbf{l}}_{\alpha,r}$ (α=0)	0,5 mm	0,5 mgon	0,1 mgon
σ	3 mm + 2 ppm	1.5 mgon	0.5 mgon

Table 1: Typical interval radii and standard deviations of the observations

	x-component	y-component
$\tilde{\hat{\mathbf{x}}}_{\alpha,r}$ (α=0)	0.2…1.0 mm	0.2…1.0 mm
σ	0.4…1.5 mm	0.4…1.5 mm

Table 2: Typical interval radii and standard deviations of the parameters

The imprecise evaluation of the parameters and the hypothesis tests is based on eleven different α-cut levels. The membership function of the imprecise regions of acceptance $\tilde{A}$ is equal to one until it reaches the fractile value $\chi^2_{0,95}(f,0)$ and then decreases linearly to zero which is reached with the fractile value $\chi^2_{0,99}(f,0)$. The influence parameters $\tilde{\mathbf{p}}$ of the sensitivity analysis are defined as triangular fuzzy intervals (linear reference functions).

The first step is the computation of the global tests for the here considered epochs 1999 and 2004. Whereas the epoch 2004 does not show any noticeable problems and the degree of rejectability for the null hypothesis (see Equation (18)) is zero, the null hypothesis for the epoch 1999 is rejected; see Figure 5 for the given test scenario and (Neumann et al. 2006) for another numerical example. The impact of remaining systematics in both global tests is small. In order to find the reason for the rejection of the null hypothesis of the global test we use one and multidimensional hypothesis testing for outlier detection, see (Neumann et al. 2006) and (Neumann and Kutterer 2006) for the test procedure. While revealing two outliers, the global test is no more rejected and the deformation analysis may start with a congruence test for the control points of the network. Note, that further reasons for the rejection of the null hypothesis of the global test can be non-suitable choices of the functional or stochastic model-components.

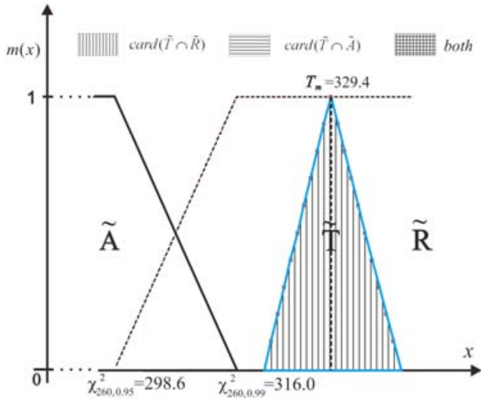

Figure 5: Global test for the epoch 1999

We carry on with the deformation analysis checking the stability of seven control points around the lock with the above-mentioned congruence test; see Fig. 6 for the imprecise test situation and cf. (Neumann and Kutterer 2006) for the evaluation of the imprecise quadratic form. Throughout this section two different cases for the congruence tests are considered: the first one is an imprecise evaluation without additional noise, it is shown on the left hand side of the below given Figures. The second one refers to the right hand side of the Figures 6-8, where the points of both epochs are afflicted with an additional Gaussian point noise of 0.25 mm in each epoch. This may be due to discretization problems of the lock and due to the object fuzziness, e. g. thermal expansion and non-controllable short-time deformation of the monitoring object. The hypotheses considered for the congruence tests are:

$$H_0: \quad \mathrm{E}(\hat{\mathbf{x}}_{m,1999c}) = \mathrm{E}(\hat{\mathbf{x}}_{m,2004c}), \tag{24\_a}$$

$$H_A: \quad \mathrm{E}(\hat{\mathbf{x}}_{m,1999c}) \neq \mathrm{E}(\hat{\mathbf{x}}_{m,2004c}), \tag{24\_b}$$

with the expected values for the control points in the epochs 1999 $E(\hat{\mathbf{x}}_{m,1999c})$ and 2004 $E(\hat{\mathbf{x}}_{m,2004c})$. Both test values are clearly inside the region of rejection and the null hypotheses are rejected. At least one point coordinate has changed significantly between the epochs 1999 and 2004.

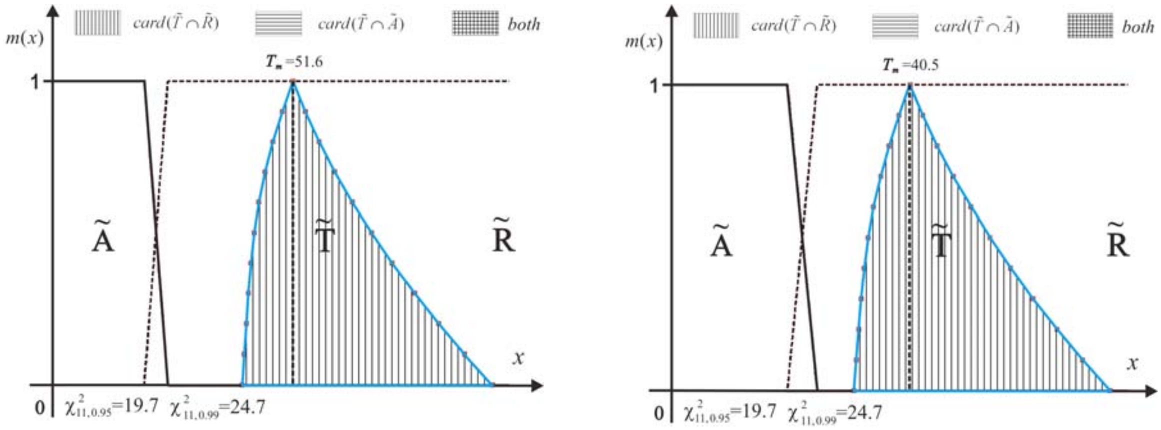

Figure 6: Congruence test with and without additional stochastic point noise

For this reason, we iterate over the number of control points to find the point, which mostly influences the quadratic test value. This procedure is based on the degree of rejectability of the null hypothesis in the imprecise case. Before applying the remaining control point to a new quadratic form, they are adjusted within a partially constrained trace minimization with respect to the remaining control points in both epochs, cf. (Welsch et al. 2000) for a detailed description. Figure 7 shows the two test situations with the lowest degree of rejectability for the null hypothesis after eliminating one control point. Whereas the congruence test without additional noise is rejected for $\rho_{crit} < 0.03$, the case with additional stochastic noise is already rejected for $\rho_{crit} < 0.52$. The mean points of the test values refer to the pure stochastic case. Due to the asymmetric test value on the right hand side of Figure 7, the degree of rejectability of the null hypothesis is higher than 0.5, although the midpoint of the test value is clearly inside the imprecise region of acceptance.

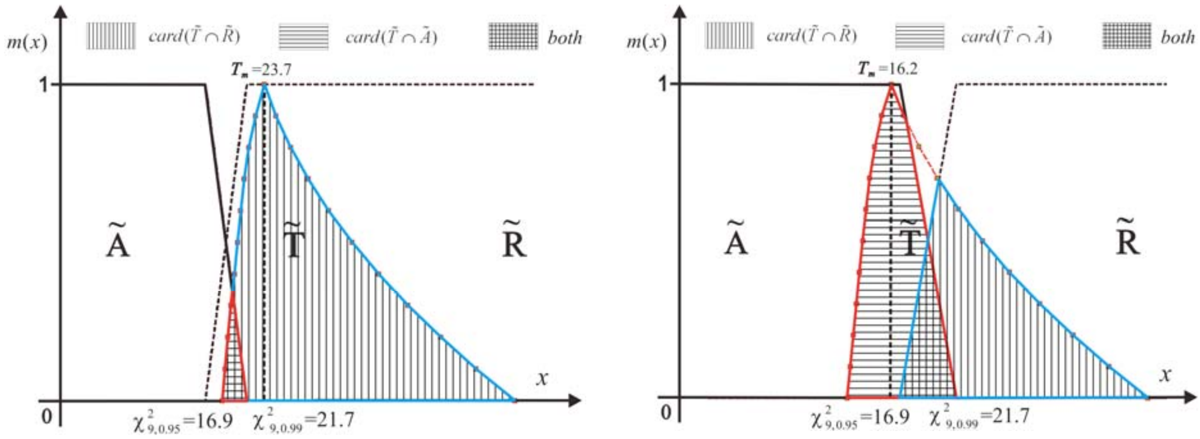

Figure 7: Congruence tests after the elimination of point 904

The final congruent point set in both epochs is obtained after eliminating the points 904 and 909 from the control points. The associated test situations are given in Figure 8. The deformation may continue with pointwise tests with the quadratic form given in Section 4 (see Equation 20) to detect movements of the object point, but this is not discussed in this paper.

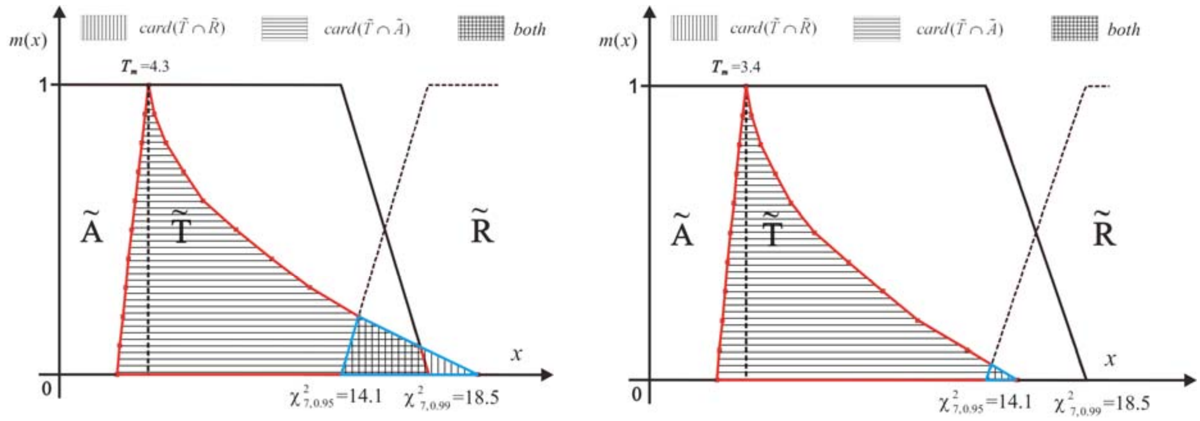

Figure 8: Congruence test after eliminating the points 904 and 909 from the control points

6. CONCLUSIONS

In this study a completely evaluated deformation analysis was presented, considering the uncertainty budget to contain two independent types of uncertainty: random variability and imprecision. The first one describes the stochastic behavior of the observations and parameters; the second one is due to remaining systematic deviations between the observations and the geodetic model. Both types of uncertainty are modeled in a comprehensive way to be more adequate in data analysis in order to avoid the quadratic propagation of remaining systematics, what leads to an underestimation of the uncertainty budget for the parameters of interest.

It is shown, that the imprecise evaluation leads to an extended uncertainty budget, noticeable by the range of the so-called fuzzy intervals. In addition, it allows computing the effects of neglected corrections or reductions on the parameters of interest. Whereas in some monitoring networks the effects of remaining systematics are unimportant, in other networks they may dominate the uncertainty budget. The extended uncertainty budget contains also some parts of the so called point noise in epoch comparison, e. g., discretization problems of the monitoring object. Furthermore, the modeled regions of transition between the regions of acceptance and rejection allow a more comprehensive test decision in hypothesis testing than in the pure stochastic case.

The presented evaluation strategy allows optimizing the network configuration with respect to random and systematic uncertainties (Schön and Kutterer 2001) and (Schön 2003). Nevertheless two further extensions have to be mentioned: the imprecise evaluation has to be extended in order to take the above-mentioned object fuzziness into account. The second extension is the development of a sensitivity analysis in the imprecise case, based on the type I and type II errors in the imprecise case, cf. (Kutterer 2004).

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