Geoid Heights Computation from GPS Data and Classical Terrestrial Zenith Angle Observations

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- This study aims the use of conventional terrestrial zenith angle and GPS data instead of GPS-geometric leveling for the determination of precise geoid heights.
- The method has been probed into in consideration of the accuracy, practicability, measurement and evaluation criteria, and has been examined.
- In addition, geoid profiles that have been determined with the GPS-Zenith(GPS_ZEN.) angles measurement have been compared with TG-99A and IGNA geoid models to explore its consistency.

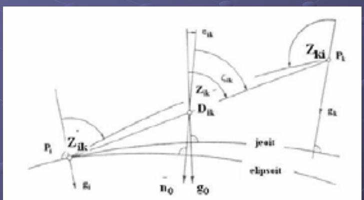
Geoid can be determined by;

- Global models, constituted from potential coefficients
- Use of vertical deflection, obtained from astro-geodetic measurement
- Gravimetric measurement,
- GPS-Geometric levelling,
- GPS-Precise trigonometric levelling,
- GPS-Astronomic observation,
- GPS- Gravimetric measurement,

MATHEMATICAL MODEL OF THE PROPOSED METHOD

$$\zeta_{ik} = \arccos((\cos\varphi_i(\cos\lambda_i \Delta X_{ik} + \sin\lambda_{ik}\Delta Y_{ik}) + \sin\varphi_i \Delta Z_{ik})/D_{ik})$$
$$\zeta_{ki} = \arccos((\cos\varphi_k(\cos\lambda_k \Delta X_{ki} + \sin\lambda_{ki}\Delta Y_{ki}) + \sin\varphi_k \Delta Z_{ki})/D_{ik}) \quad (1)$$
$$D_{ik} = \sqrt{(\Delta X_{ik}^2 + \Delta Y_{ik}^2 + \Delta Z_{ik}^2)}$$

$$\zeta_0 = \arccos((\cos\varphi_0(\cos\lambda_0 \Delta X_{ik} + \sin\lambda_0\Delta Y_{ik}) + \sin\varphi_0 \Delta Z_{ik})/D_{ik}) \quad (2)$$
$$\varphi_0 = \frac{\varphi_i + \varphi_k}{2}, \lambda_0 = \frac{\lambda_i + \lambda_k}{2} \quad (3)$$



$$Z_0 = \frac{(Z_{ik} + 200^g - Z_{ki})}{2} \quad (4)$$

$$\varepsilon_0 = Z_0 - \zeta_0 \quad (5)$$

$$\Delta N_{ik} = D_{ik} \sin Z_0 \sin \varepsilon_{ik} \quad (6)$$

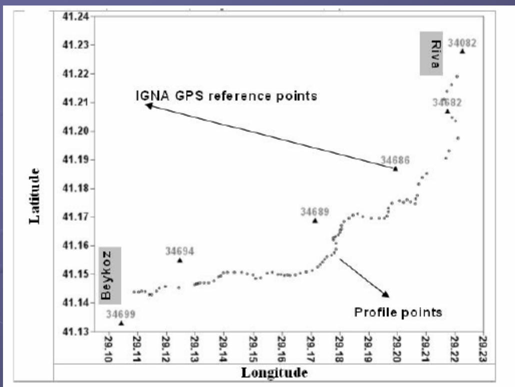


Figure.2. View of the profiles in horizontal plane

Table.1. Surveying configuration of GPS network

Survey mode: Static observation with 2 referance on IGNA points, third receiver rover on the points of the profile			
1. Reference point of loop	2. Reference point of loop	Measured Profile points	Number of loop for each campaign
34699	34694	1-10	10
34694	34689	11-43	33
34689	34686	44-70	27
34686	34682	71-81	11
34682	34082	82-87	6
Session interval: 30 minute Elevation mask: 15° Epoch: 10 second Number of receiver: 3 receiver with geodetic antenna			

Table.2. Processing and adjustment informations of GPS network

GPS Network informations	
Number of fixed points (XYZ fixed)	6
Number of total points (k)	93
Number of baseline (q)	174
Number of baseline components (n=3q)	522
Standard deviation of baseline components	
Distance of baseline (m)	Max.=0.017, Min.=0.004, RMS=0.010
ΔX Components (m)	Max.=0.010, Min.=0.001, RMS=0.006
ΔY Components (m)	Max.=0.012, Min.=0.001, RMS=0.005
ΔZ Components (m)	Max.=0.011, Min.=0.002, RMS=0.006
Adjustment informations	
Number of observation (n+f)	522
Number of parameters (u=3k)	261
Degree of freedom (n+f-u)	261
Unit weighted variance	7.21
Sum of the variance (V^TPV)	1880.00
Residuals	
V (radial) residuals (m)	Max.=0.039, Min.=0.001, RMS=0.015
$V_{\Delta X}$ residuals (m)	Max.=0.027, Min.=-0.024 ,RMS=0.009
$V_{\Delta Y}$ residuals (m)	Max.=0.039, Min.=-0.027 ,RMS=0.009
$V_{\Delta Z}$ residuals (m)	Max.=0.023, Min.=-0.018 ,RMS=0.008
Coordinate errors	
Latitude errors (m)	Max.=0.014,Min.=0.003,RMS=0.009
Longitude errors (m)	Max.=0.011,Min.=0.004,RMS=0.007
Elipsoidal Height errors (m)	Max.=0.013,Min.=0.005,RMS=0.009

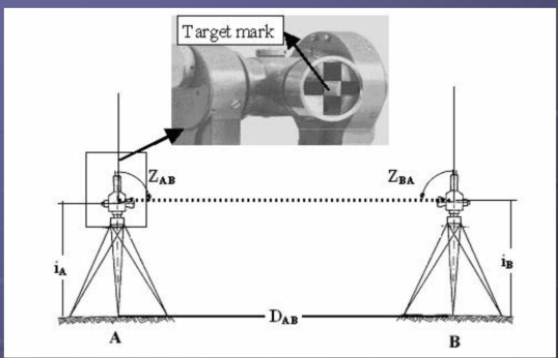


Table.3. Geoid height computation between point 1 to 11

NO	D_{ik}(m)	$Z_{0(grad)}$	$\xi_{0(grad)}$	ΔN_{ik}(m)	N(m)	N_{IGNA}(m)	N_{TG99A}(m)	N-N_{IGNA}	N-N_{TG99A}
34699					*36,685*				
1	1772,850	97,09690	97,10242	-0,154	36,530	36,544	36,589	-0,014	-0,059
2	129,446	100,40079	100,40395	-0,006	36,521	36,541	36,587	-0,020	-0,066
3	98,695	101,17529	101,18245	-0,011	36,509	36,539	36,586	-0,030	-0,077
4	102,280	100,81475	100,81261	0,003	36,510	36,538	36,584	-0,028	-0,074
5	159,527	100,87036	100,86897	0,004	36,512	36,539	36,582	-0,027	-0,070
6	83,700	100,51793	100,51082	0,009	36,519	36,538	36,581	-0,019	-0,062
7	202,336	100,38319	100,38277	0,001	36,519	36,531	36,578	-0,012	-0,059
8	135,025	101,15139	101,14384	0,016	36,533	36,527	36,575	0,006	-0,042
9	191,992	101,05678	101,05688	0,000	36,531	36,522	36,570	0,009	-0,039
11	385,912	100,91275	100,91210	0,004	36,533	36,517	36,562	0,016	-0,029
34694	588,883	109,82263	109,81631	0,039	*36,589*				
				-0,075	**-0,096**				

Table 4. Geoid height computation between point 13 to 45

NO	D_{ik}(m)	$Z_{0(grad)}$	$\xi_{0(grad)}$	ΔN_{ik}(m)	N(m)	N_{IGNA}(m)	N_{TG99A}(m)	N-N_{IGNA}	N-N_{TG99A}
34694					*36,589*				
13	443,654	88,37423	88,38627	-0,084	36,507	36,506	36,550	0,001	-0,043
14	56,319	100,63299	100,63355	-0,001	36,507	36,505	36,548	0,002	-0,041
15	52,040	99,71982	99,70575	0,012	36,519	36,503	36,547	0,016	-0,028
16	70,479	101,64229	101,65178	-0,011	36,509	36,501	36,545	0,008	-0,036
17	106,704	100,91522	100,91543	0,003	36,513	36,500	36,543	0,013	-0,030
18	129,075	101,69880	101,69337	0,011	36,524	36,498	36,540	0,026	-0,016
19	176,920	101,76014	101,76158	-0,004	36,521	36,493	36,536	0,028	-0,015
20	204,038	101,77429	101,77319	0,004	36,525	36,486	36,531	0,039	-0,006
21	79,400	102,20774	102,20975	-0,003	36,523	36,484	36,530	0,039	-0,007
22	136,634	102,63788	102,62925	0,019	36,542	36,479	36,527	0,063	0,015
23	131,973	103,04354	103,05127	-0,016	36,526	36,477	36,524	0,049	0,002
24	179,326	103,58850	103,58690	0,005	36,531	36,474	36,520	0,057	0,011
25	198,829	103,36251	103,35834	0,013	36,545	36,471	36,516	0,074	0,029
26	178,892	106,31311	106,31740	-0,012	36,533	36,470	36,513	0,063	0,020
27	168,704	107,60292	107,60425	-0,004	36,530	36,469	36,510	0,061	0,020
28	152,216	106,45937	106,45748	0,005	36,535	36,471	36,511	0,064	0,024
29	147,608	108,50179	108,50288	-0,003	36,534	36,469	36,507	0,065	0,027
30	258,052	94,71695	94,71620	0,003	36,537	36,460	36,501	0,077	0,036
31	164,152	97,37326	97,37617	-0,008	36,530	36,457	36,497	0,073	0,033
32	173,557	97,07842	97,07824	0,001	36,531	36,457	36,494	0,074	0,037
33	80,031	96,54513	96,53995	0,007	36,538	36,455	36,492	0,083	0,046
34	109,133	98,21470	98,21936	-0,008	36,531	36,455	36,490	0,076	0,041
35	93,103	95,41541	95,42346	-0,012	36,519	36,433	36,487	0,066	0,032
36	67,068	96,76935	96,76459	0,005	36,525	36,453	36,486	0,072	0,039
37	169,671	97,25671	97,25314	0,010	36,535	36,450	36,481	0,085	0,054
38	153,416	98,08337	98,09437	-0,027	36,509	36,445	36,476	0,064	0,033
39	174,305	98,19437	98,19565	-0,004	36,506	36,442	36,470	0,064	0,036
40	184,430	98,61982	98,62862	-0,026	36,481	36,437	36,464	0,044	0,017
41	206,493	98,32787	98,32386	0,007	36,488	36,450	36,456	0,038	0,032
42	136,322	98,92349	98,93353	-0,022	36,467	36,426	36,452	0,041	0,015
43	111,381	99,48040	99,47583	0,008	36,475	36,422	36,449	0,053	0,026
44	136,079	98,22432	98,22876	-0,010	36,466	36,418	36,445	0,048	0,021
45	111,318	98,26119	98,27978	-0,033	36,434	36,414	36,441	0,020	-0,007
34689	844,969	105,95807	105,96216	-0,054	*36,583*				
				-0,226	**-0,208**				

Table.5. Geoid height computation between point 46 to 74

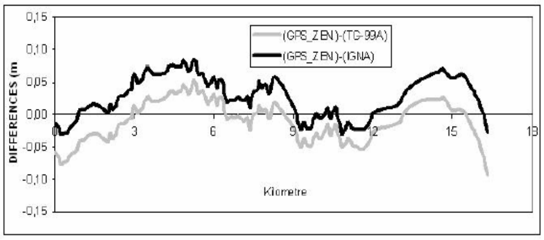
NO	D_{ik}(m)	$Z_{0(grad)}$	$\xi_{0(grad)}$	ΔN_{ik}(m)	N(m)	N_{IGNA}(m)	N_{TG99A}(m)	N-N_{IGNA}	N-N_{TG99A}
34689					*36,381*				
46	801,034	93,69239	93,68823	0,052	36,432	36,409	36,436	0,023	-0,004
47	136,920	98,61707	98,61928	-0,005	36,426	36,405	36,431	0,021	-0,005
48	140,894	99,89649	99,89572	0,002	36,427	36,400	36,428	0,027	-0,001
49	229,245	98,53384	98,53752	-0,013	36,413	36,392	36,423	0,021	-0,010
50	151,877	97,53884	97,53628	0,006	36,418	36,389	36,423	0,029	-0,005
51	69,279	99,80239	99,82054	-0,020	36,397	36,387	36,422	0,010	-0,025
52	82,017	98,96524	98,95398	0,015	36,411	36,384	36,419	0,027	-0,008
53	110,014	100,41603	100,41227	0,007	36,416	36,380	36,415	0,036	0,001
54	92,715	98,96377	98,96446	-0,001	36,415	36,377	36,412	0,038	0,003
55	86,018	97,55258	97,54370	0,012	36,426	36,374	36,411	0,052	0,015
56	80,060	94,17833	94,18791	-0,012	36,413	36,371	36,408	0,042	0,005
57	129,971	94,84490	94,84368	0,003	36,414	36,367	36,405	0,047	0,009
58	159,486	99,93330	99,94089	-0,019	36,394	36,361	36,400	0,033	-0,006
59	197,644	101,42249	101,41637	0,019	36,412	36,354	36,394	0,058	0,018
60	169,571	98,26939	98,27352	-0,011	36,400	36,348	36,388	0,052	0,012
61	148,559	96,53102	96,53617	-0,012	36,387	36,344	36,383	0,043	0,004
62	178,976	98,99614	99,00005	-0,011	36,375	36,346	36,381	0,029	-0,006
63	256,480	99,11753	99,12324	-0,023	36,352	36,344	36,375	0,008	-0,023
64	219,461	98,18699	98,19671	-0,034	36,317	36,341	36,369	-0,024	-0,052
65	167,143	99,86932	99,86532	0,011	36,327	36,338	36,364	-0,011	-0,037
66	89,456	97,93152	97,93971	-0,012	36,314	36,335	36,361	-0,021	-0,047
67	173,088	99,09735	99,09882	-0,004	36,309	36,329	36,357	-0,020	-0,048
68	155,177	99,31360	99,30683	0,017	36,325	36,324	36,354	0,001	-0,029
69	203,807	99,91344	99,91750	-0,013	36,311	36,317	36,347	-0,006	-0,036
70	244,706	100,18624	100,18390	0,009	36,319	36,309	36,334	0,010	-0,015
72	125,175	102,67387	102,68150	-0,015	36,303	36,310	36,337	-0,007	-0,034
74	152,171	97,14287	97,13659	0,015	36,317	36,305	36,332	0,012	-0,015
34686	627,008	105,51981	105,52348	-0,036	*36,280*				
				-0,074	**-0,101**				

Table.6. Geoid height computation between point 75 to 89

NO	D_{ik}(m)	$Z_{0(grad)}$	$\xi_{0(grad)}$	ΔN_{ik}(m)	N(m)	N_{IGNA}.(m)	N_{TG99A}.(m)	$N-N_{IGNA}$	$N-N_{TG99A}$
34686					*36,280*				
75	725,172	95,43349	95,43195	0,018	36,299	36,306	36,331	-0,007	-0,032
77	152,889	98,10925	98,11925	-0,024	36,277	36,307	36,329	-0,030	-0,052
78	198,968	98,28797	98,28493	0,010	36,288	36,300	36,324	-0,012	-0,036
79	141,507	99,84790	99,85443	-0,015	36,275	36,295	36,320	-0,020	-0,045
81	430,645	99,20891	99,21209	-0,022	36,255	36,279	36,308	-0,024	-0,053
82	258,094	100,05472	100,05411	0,003	36,259	36,269	36,299	-0,010	-0,040
83	206,992	99,92953	99,92753	0,007	36,267	36,262	36,291	0,005	-0,024
87	817,938	100,29861	100,30013	-0,020	36,249	36,234	36,262	0,015	-0,013
88	303,344	99,68264	99,68096	0,008	36,258	36,223	36,253	0,035	0,005
89	565,728	100,36130	100,36147	-0,002	36,258	36,203	36,237	0,055	0,021
34682	457,845	105,10638	105,11872	-0,089	*36,171*				
				-0,125	**-0,109**				

Table.7. Geoid height computation between point 91 to 100

NO	D_{ik}(m)	$Z_{0(grad)}$	$\xi_{0(grad)}$	ΔN_{ik}(m)	N(m)	N_{IGNA}.(m)	N_{TG99A}.(m)	$N-N_{IGNA}$	$N-N_{TG99A}$
34682					*36,171*				
91	202,167	101,25995	101,23491	0,080	36,248	36,182	36,224	0,066	0,024
92	176,266	98,16136	98,16045	0,003	36,249	36,179	36,223	0,070	0,026
93	298,854	95,38018	95,38392	-0,018	36,229	36,172	36,220	0,057	0,009
94	442,416	96,95554	96,95641	-0,006	36,221	36,159	36,214	0,062	0,007
95	329,252	99,36015	99,36585	-0,030	36,189	36,149	36,207	0,040	-0,018
96	269,332	100,55851	100,56466	-0,026	36,161	36,139	36,201	0,022	-0,040
100	356,534	100,21013	100,22048	-0,058	36,101	36,127	36,194	-0,026	-0,093
34082	577,755	103,76598	103,76889	-0,026	*36,072*				
				-0,081	**-0,099**				



	(GPS_ZEN)-(TG-99A)	(GPS_ZEN.)-(IGNA)
Absolute value of the Minimum difference (m)	0.001	0.001
Absolute value of the Maximum difference (m)	0.093	0.085
Avarege of absolute difference (m)	0.028	0.036
Standart deviataion of average of difference (m)	0.020	0.023
RMS value of the difference (m)	0.034	0.043

Figure.7. Comparation of the (GPS_ZEN) with IGNA and TG-99A model

- In length of sight shorter than 500 meters, one may accept that the effect of deviation of verticle on height is negligibly small. In this regard, to determine geoid heights by using proposed models zenith angle measurements are made with short lengths of sight. The most important remaining effect is the refraction effect, by choosing the length of sights between points short, and by making simultaneous and reciprocal zenith angle observations by using special equipment in favorable meteorological conditions, this effect may be reduced to a great extent.
- When appropriate measurement and processing strategies are applied with the GPS, it is possible to determine ellipsoidal heights with an accuracy of about 9mm (Max. 13mm).
- Consistency of geoid profile determined with the GPS_ZEN. with the IGNA geoid is 43mm (Max. 85mm, Min. 1mm), while its consistency with the TG-99A geoid is 34mm(Max. 93mm, Min. 1mm).

Thanks for your attention !