Optimized Kalman Filter versus Rigorous Method in Deformation Analysis

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ABSTRACT:

Kalman filtering is a multiple-input, multiple-output filter that can optimally estimate the states of a system, so it can be considered a suitable means for deformation analysis. The states are all the variables needed to completely describe the system behavior of the deformation process as a function of time (such as position, velocity etc.). The standard Kalman filter estimates the state vector where the measuring process is described by a linear system. While, in order to process a non-linear system an optimized aspect of Kalman filter is appropriate. The main purpose of this research is to evaluate the Optimized Kalman filter as a non-robust method versus the IWST (Iterative Weighted Similarity Transformation) as a rigorous (also called robust) method. To satisfy this objective, first a detailed description on executing the Extended Kalman filter using the observation of angles and distances directly is provided. Later on, five sets of 2-D Total Station data include distances and angles are used to demonstrate the Optimized Kalman Filter. For detecting the deformation, single point test for every point is applied component by component as a local test. Later on, the findings from Optimized Kalman Filter are compared and evaluated against the results from IWST testing. In general, the outcome of Kalman filter algorithm is close to the preliminary results from IWST testing. The maximum and minimum differences in computed displacements are equal to 0.0002 and 0.002 in meters respectively. Finally, Kalman filter approaches, having some properties, are recognized as suitable techniques for deformation analysis.

1. INTRODUCTION

1.1 Introduction

In consequence of some factors either natural (e.g. water pressure and earthquake) or artificial (e.g. the weight of the structure itself), the deformable object is subjected to vary from normal position and dimension. Deformation monitoring refers to a regular observation of the alterations of a deformable object, whereas deformation analysis is the task of processing the available data from monitoring phase and detecting the magnitude and location of the displacement (Chrzanowski et al., 2007; USACE, 2002).

There are several recognized data processing techniques which are categorized into two main classes, i.e. Robust methods, such as IWST (Iterative Weighted Similarity Transformation) and Non-robust methods, such as Kalman Filtering and etc. (Tasci, 2010). The ordinary Kalman Filter combines all the available information to optimize the estimated state vector minimizing estimated error covariance where the measuring process is described by a linear system. In this study, among the various identified data processing techniques the suitability of the Optimized Kalman Filter (OKF) for deformation analysis when the observations are applied directly and the stochastic difference equation and/or the measurement equation are non-linear is evaluated. In order to illustrate the OKF five sets of 2-D Total Station data which includes distances and angles are used.

For detecting the deformation, the single point test is applied as a local test. Being different from the ordinary single point test (Md Som et al., 2004; Lim et al., 2010) of the single point test which has been used in Ince and Sahin (2000), the single point test used in this study examine every components of the position of each points. For every component if $T\text{(computed)} \geq F\left(1-\alpha, t, f_0\right)\text{(from table)}$, the relevant point is consider as unstable point (Ince and Sahin, 2000).

2. KALMAN FILTER

2.1 Ordinary Kalman filter

In 1960, R.E. Kalman introduced his new approach which is a powerful optimal recursive process to filtering, prediction and smoothing the parameters which vary with time (Welch and Bishop, 2006; Cross, 1983). This approach was first used in electrical control systems (Kalman, 1960). It has also been used as one of the methods used in deformation analysis. The ordinary Kalman filter also called “Classical Kalman filter” or “Discrete Kalman filter” is a suitable tool for deformation analysis where the measuring process is able to be governed by a linear system. The main equations of ordinary Kalman filter are detailed in Aharizad and Setan (2011). The Optimized Kalman filter is recommended where the distance and angle measurements are directly processed, the process to be estimated and (or) the measurement relationship to the process is non-linear (Welch and Bishop, 2006).

2.2 Optimized Kalman filter

Assume the non-linear function f relates the state at the earlier time step $i-1$ to the state at the present time step i (Welch and Bishop, 2001):

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$$x_i = f\left(x_{i-1}, u_i, w_{i-1}\right) \quad (1)$$

and the non-linear function h relates the state x_i to the measurement z_i:

$$z_i = h\left(x_i, v_i\right) \quad (2)$$

In practice the exact values of the noise w_i and v_i at each time step may not be known. So, the approximation of the state and measurement vector will be used without them as:

$$x_i = f\left(x_{i-1}, u_i, 0\right) \quad (3)$$

and the non-linear function h relates the state x_i to the measurement z_i:

$$z_i = h\left(x_i, 0\right) \quad (4)$$

Then the EKF Prediction equations or the time update equations, and the filtering equations, also called the measurement update equations are given as in Eq (5) to Eq (9) in that order (Welch and Bishop, 2001):

$$\hat{x}_i = f\left(\hat{x}'_{i-1}, u_i, 0\right) \quad (5)$$

$$C_{\hat{x}_i} = M_{i-1,i} C_{\hat{x}'_{i-1}} M^T_{i-1,i} + W_i Q_{i-1} W_i^T \quad (6)$$

$$G_i = C_{\hat{x}_i} A_i^T \left(A_i C_{\hat{x}_i} A_i^T + V_i R_i V_i^T\right)^{-1} \quad (7)$$

$$\hat{x}'_i = \hat{x}_i + G_i\left(z_i - h\left(\hat{x}_0^-, 0\right)\right) \quad (8)$$

$$C_{\hat{x}'_i} = \left[I - G_i A_i\right] C_{\hat{x}_i} \quad (9)$$

Then the *A*, *W*, *M* and *V* matrices can be derived using partial derivatives as:

$$A_{[i,j]} = \frac{\partial f_{[i]}}{\partial x_{[j]}}\left(\hat{x}_{i-1}, u_i, 0\right) \quad (10)$$

$$W_{[i,j]} = \frac{\partial f_{[i]}}{\partial w_{[j]}}\left(\hat{x}_{i-1}, u_i, 0\right) \quad (11)$$

$$M_{[i,j]} = \frac{\partial h_{[i]}}{\partial x_{[j]}}\left(\hat{x}_i, 0\right) \quad (12)$$

$$V_{[i,j]} = \frac{\partial h_{[i]}}{\partial v_{[j]}}\left(\hat{x}_i, 0\right) \quad (13)$$

Note that for simplicity in the notation we do not use the time step subscript i with the Jacobians *A*, *W*, *M* and *V*, even though they are in fact different at each time step.
Where:

x_i and z_i = The state and measurement vectors at epoch i respectively;

u_i = The optional control input;

w_i = The process noise;

v_i = The measurement noise;

$\hat{x}_i$ = The predicted state vector also called the a priori state estimate at epoch i;

$\hat{x}'_i$ = The filtered state vector also called the a posteriori state estimate at epoch i;

$C_{\hat{x}_i}$ = The a priori estimate error covariance at epoch i;

$C_{\hat{x}'_i}$ = The a posteriori estimate error covariance at epoch i;

Q_i = The process noise covariance at epoch i;

G_i = The Kalman gain matrix at epoch i;

R_i = The measurement noise covariance at epoch i.

3. STATISTICAL TEST AND DEFORMATION DETECTION

The single point test as the local statistical test is carried out for each point component by component at the significance level $\alpha = 0.05$. The critical value is computed via Eq. (14) to Eq. (18) (Ince and Sahin, 2000; Aharizad and Setan, 2011).

$$\begin{cases} dx_{i-1,i} = x_i - x_{i-1} \\ dy_{i-1,i} = y_i - y_{i-1} \end{cases} \quad (14)$$

$$dv_{i-1,i} = \sqrt{\left(dx_{i-1,i}\right)^2 + \left(dy_{i-1,i}\right)^2} \tag{15}$$

The zero and alternative hypotheses are:

$$\begin{cases} H_0 : E\{d^x_{i-1,i}\} = 0 \\ H_0 : E\{d^y_{i-1,i}\} = 0 \end{cases} , \quad \begin{cases} H_a : E\{d^x_{i-1,i}\} \neq 0 \\ H_a : E\{d^y_{i-1,i}\} \neq 0 \end{cases} \tag{16}$$

$$\begin{bmatrix} \sigma^2 d^x_{i-1,i} \\ \sigma^2 d^y_{i-1,i} \end{bmatrix} = \begin{bmatrix} \sigma^i_{xx} \\ \sigma^i_{yy} \end{bmatrix} + \begin{bmatrix} \sigma^{i-1}_{xx} \\ \sigma^{i-1}_{yy} \end{bmatrix} \tag{17}$$

$$\begin{bmatrix} T^x_{i-1,i} \\ T^y_{i-1,i} \end{bmatrix} = \begin{bmatrix} \dfrac{\left(d^x_{i-1,i}\right)^2}{\sigma^2 d^x_{i-1,i}} \\ \dfrac{\left(d^y_{i-1,i}\right)^2}{\sigma^2 d^y_{i-1,i}} \end{bmatrix} \tag{18}$$

Where $dx_{i-1,i}$ and $dy_{i-1,i}$ = the x (East) and y (North) components of displacement vector at epoch i respectively.

$\sigma^2_{d^x_{i-1,i}}$ and $\sigma^2_{d^y_{i-1,i}}$ = The variance of the difference vector for x and y components respectively;

σ^{i-1}_{xx} and σ^{i}_{xx} = The variance of the x estimation at $i-1$ and i epochs respectively;

σ^{i-1}_{yy} and σ^{i}_{yy} = The variance of the x estimation at $i-1$ and i epochs respectively;

The computed $T^x_{i-1,i}$ and $T^y_{i-1,i}$ values are compared with critical value derived from t-test table based on the α and degrees of freedom f_0. For every point if either $T^x_{i-1,i} \geq F(1-\alpha, t, f_0)$ or $T^y_{i-1,i} \geq F(1-\alpha, t, f_0)$, it is considered that the difference vector, $dv_{i-1,i}$, is significant and the relevant point is segregated as unstable point.

4. S-TRANSFORMATION

The method of S-transformation is applied to avoid the problem of datum dependent displacement and to make the results comparable with the preliminary results from IWST testing. For this purpose the five control stations (point 1 to point5) are taken into account as the datum points and partial minimum trace technique is performed (Setan, 1995). So, the relevant components of matrix W (Eq (19)) to the five mentioned control stations are all one.

$$S = I - G\left(G^T\ W\ G\right)^{-1} G^T\ W \tag{19}$$

$$\begin{cases} x_j = S\ x_i \\ Q_{x_j} = S\ Q_{x_i} S^T \end{cases} \tag{20}$$

Generally, G^T in Eq (19) for 2-D network is defined as:

$$G^T = \begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 1 \\ y^0_1 & -x^0_1 & y^0_2 & -x^0_2 & \cdots & y^0_n & -x^0_n \\ x^0_1 & y^0_1 & x^0_1 & y^0_1 & \cdots & x^0_n & y^0_n \end{bmatrix} \tag{21}$$

Where first two rows of G^T define the translation along x and y axes respectively. The third and fourth rows are related to the rotations around the z axis and scale of the network respectively. Since, in this study the observed data includes distance and bearing the third and fourth rows are eliminated (Setan, 1995).

In Eq (3.33), n represents number of stations and:

$$\begin{cases} x^0_i = x_i - \dfrac{\left(\sum_{i=1}^{n} x_i\right)}{n} \quad \& \quad y^0_i = y_i - \dfrac{\left(\sum_{i=1}^{n} y_i\right)}{n} \end{cases} \tag{22}$$

5. DATA ANALYSIS AND RESULTS

5.1 Total Station Data

In this study, five sets of Total Station data which were acquired from monitoring of a concrete block subjected to load is used in the deformation process directly. A micro triangulation network consists of 12 stations, 5 reference points and 7 object points, was established inside a laboratory. Points 1 to 5 were located within an area of about 6x4 m^2 as the reference points (Figure 1). The object points, points 6 to 12, were sited on one facade of the block (Figure 2). A 2-D triangulation measurement of distance and bearing were done using Sokkia Set3 Total Station (Md Som et al., 2004).

As data includes distances and azimuths the system models used as primary and secondary models will be non-linear. So, Optimized Kalman filter approach has to be used to analyze the data and compute the deformation vector.

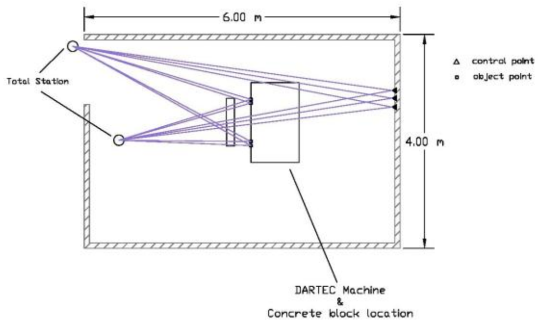

Figure 1. Micro triangulation monitoring network set up in the laboratory (Md Som et al., 2004)

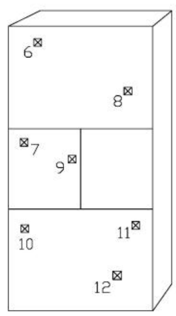

Figure 1. Object points location on the facade of conceret block (Md Som et al., 2004)

5.2 Data Processing Steps

The overall of data processing is fulfilled in four main steps:

1. Kalman filter executing;
2. Deformation detection using Kalman filter outcomes directly;
3. S-transformation;
4. Deformation detection using the outcomes of S-transformation phase.

The Kalman filter step by itself contains six modules:

1. Observation vector (b_i) introducing (Eq. (23));
2. State vector (X_i) definition (Eq. (24));
3. Observation (primary) models definition (Eq. (25) to Eq. (28));
4. Dynamic (secondary) models definition (Eq. (29) to Eq. (32);

$$b_1 = \begin{bmatrix} D1,2 \\ D1,3 \\ D1,4 \\ \vdots \\ Az1,2 \\ Az1,3 \\ Az1,4 \\ \vdots \\ a_{x_2} \\ a_{y_2} \\ \vdots \end{bmatrix}_1 , b_2 = \begin{bmatrix} D1,2 \\ D1,3 \\ D1,4 \\ \vdots \\ Az1,2 \\ Az1,3 \\ Az1,4 \\ \vdots \\ a_{x_2} \\ a_{y_2} \\ \vdots \end{bmatrix}_2 , \ldots , b_i = \begin{bmatrix} D1,2 \\ D1,3 \\ D1,4 \\ \vdots \\ A1,2 \\ A1,3 \\ Az1,4 \\ \vdots \\ a_{x_2} \\ a_{y_2} \\ \vdots \end{bmatrix}_i \quad (23)$$

$$X_i = \begin{bmatrix} x_2 \\ y_2 \\ x_3 \\ y_3 \\ \vdots \\ v_{x_2} \\ v_{y_2} \\ \vdots \\ v_{x_{12}} \\ v_{y_{12}} \end{bmatrix}_i \quad (32)$$

Distance relates to position model:

$$f_D : D_{ij} = \sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2} \quad (25)$$

Bearing relates to position model:

$$f_{Az} : Az_{ij} = \tan^{-1} \frac{\left(x_j - x_i\right)}{\left(y_j - y_i\right)} \quad (26)$$

Acceleration relates to velocity models:

$$f_{v_x} : a_X = \frac{v_x}{dt} \quad (27)$$

$$f_{v_y} : a_y = \frac{v_y}{dt} \quad (28)$$

Relates position at epoch i to epoch i-1:

$$h_{x_i} : x_i = x_{i-1} + v_{x_{i-1}} dt + \frac{1}{2} a_X \, dt^2 \quad (29)$$

$$h_{y_i} : y_i = y_{i-1} + v_{y_{i-1}} dt + \frac{1}{2} a_Y \, dt^2 \quad (30)$$

Relates velocity at epoch i to epoch i-1:

$$h_{v_{x_i}} : v_{x_i} = v_{x_{i-1}} + a_x \, dt \tag{31}$$

$$h_{v_{y_i}} : v_{y_i} = v_{y_{i-1}} + a_y \, dt \tag{32}$$

5. Design matrix (A) and state transition matrix (M) derivation or Primary and Secondary models linearization;

Matrix *A* is computed by first order differentiation of primary model with respect to elements of state vector (Eq. (10)). Matrix $M_{i-1,i}$ is derived by first order differentiation of secondary model with respect to elements of state vector (Eq. (12)) and secondary model is changed into matrix form

$$\begin{bmatrix} x_2 \\ y_2 \\ x_3 \\ y_3 \\ \vdots \\ v_{x_2} \\ v_{y_2} \\ \vdots \\ v_{x_{12}} \\ v_{y_{12}} \end{bmatrix}_i = M_{i-1,i} \begin{bmatrix} x_2 \\ y_2 \\ x_3 \\ y_3 \\ \vdots \\ v_{x_2} \\ v_{y_2} \\ \vdots \\ v_{x_{12}} \\ v_{y_{12}} \end{bmatrix}_{i-1} + T \begin{bmatrix} a_x \\ a_y \end{bmatrix} \tag{33}$$

6. Kalman initial value computation;
7. Prediction step (Eq. (5) and Eq. (6));
8. Filtering step (Eq. (7) to Eq. (9)).

5.3 Results

Due to the page limitation in this article, only the results of first two epochs are shown in this article. Table 1 shows the total displacement of each point before applying the S-transformation (case A) along with the point status resulting from the statistical test phase. Whereas, the outcome of test statistic and deformation analysis after implementation of s-transformation (case B) that identifies whether a point remains stable or not is given in Table 2. The results shown in Table 2 are comparable with the preliminary results of IWST testing which are represented in Table 3.

Table 4 and Table 4 stand for the details of single point test for both cases (case A and case B) respectively.

Station	Kalman filter (before S-transformation) (A)	
	Displacement	status
1	0 m	stable
2	0.000001 m	stable
3	0.0001 m	stable
4	0.0001 m	stable
5	0.0001 m	stable
6	0.0009 m	stable
7	0.0026 m	moved
8	0.0043 m	moved
9	0.0048 m	moved
10	0.0014 m	moved
11	0.0008 m	stable
12	0.0062 m	moved

Table 1. Displacement and point status before applying the S-transformation

Station	Kalman filter (after S-transformation) (B)	
	Displacement	status
1	0.0001 m	stable
2	0.0001 m	stable
3	0.0004 m	stable
4	0.0001 m	stable
5	0.0003 m	stable
6	0.0012 m	moved
7	0.0028 m	moved
8	0.0044 m	moved
9	0.0049 m	moved
10	0.0016 m	moved
11	0.0008 m	moved
12	0.0064 m	moved

Table 2. Displacement and point status after applying the S-transformation

Station	IWST testing	
	Displacement	status
1	0.0014 m	stable
2	0.0008 m	stable
3	0.0008 m	stable
4	0.0007 m	stable
5	0.0006 m	stable
6	0.0010 m	moved
7	0.0025 m	moved
8	0.0041 m	moved
9	0.0046 m	moved
10	0.0012 m	moved
11	0.0012 m	moved
12	0.0044 m	moved

Table 3. Preliminary results of IWST testing

Station	Test statistic (case A)		Critical value	Status
	x component	y component	1.6762	stable
2	0.000001	0.000001	1.6762	stable
3	0.0072	0.0059	1.6762	stable
4	0.000001	0.0055	1.6762	stable
5	0.0317	0.0126	1.6762	stable
6	1.4741	0.6464	1.6762	moved
7	6.6794	6.2047	1.6762	moved
8	14.8087	17.2527	1.6762	moved
9	9.5884	25.8852	1.6762	moved
10	1.0381	1.9506	1.6762	moved
11	0.9712	0.5818	1.6762	moved
12	0.8108	2.7692	1.6762	moved

Table 4. Details of single point test (case A)

Station	Test statistic (case A)		Critical value	Status
	x component	y component	1.6762	stable
1	0.0121	0.0103	1.6762	stable
2	0.1059	0.0930	1.6762	stable
3	0.0039	0.0099	1.6762	stable
4	0.1079	0.0056	1.6762	stable
5	0.2026	0.7036	1.6762	stable
6	6.3055	15.0574	1.6762	moved
7	15.1628	106.34	1.6762	moved
8	41.2896	461.4265	1.6762	moved
9	35.5488	413.0073	1.6762	moved
10	2.8327	220.48	1.6762	moved
11	2.9140	34.1443	1.6762	moved
12	0.8786	2.9173	1.6762	moved

Table 5. Details of single point test (case B)

Additionally, the state vector includes the velocity components. So the variation of displacement is computed in both x and y directions. Table 6 gives the idea about position variations between first and second epochs in x and y directions (Vx and Vy in that order).

station	Vx (m/s)	Vy (m/s)
1	assumed as fix	assumed as fix
2	-0.00022	0.00023
3	-0.00025	0.00027
4	-0.00021	0.00029
5	-0.00009	0.00015
6	0.00003	0.00002
7	0.00017	-0.00030
8	0.00033	-0.00060
9	0.00026	-0.00075
10	0.00001	-0.00009
11	0.00001	0.00033
12	0.00038	-0.00115

Table 6. Rate of variation

6. CONCLUSIONS

The main objective of this study was the evaluating of Optimized Kalman Filter implementation and suitability in deformation analysis against a rigorous method (IWST). To satisfy this objective five set of 2-D data was utilized to execute Kalman Filter in kinematic mode.

The results of Kalman Filter approach are verified with the preliminary results of IWST. Being different from IWST method of Kalaman Filtering, Kalman Filtering combines all the information to estimate the desired parameters. A simple single point test can be applied component by component to detect both the stable and unstable points. The elements of the state vector are position and the variation of the position. Hence, Kalman Filtering is suitable for study the behavior of the deformation for investigation of catastrophes.

REFERENCES

Aharizad, N. and Setan, H., 2011. Multidimensional deformation analysis with kalman filter. 11th South East Asian Survey Congress. Co-sponsored by FIG. 22-24 June. Kuala Lumpur, Malaysia.

Chrzanowski, A., Szostak-Chrzanowski, A., Bond, J., and Wilkins, R., 2007. Increasing public and environmental safety through integrated monitoring and analysis of structural and ground deformations. *Geomatics Solutions for Disaster Management (eds: J. Li, S. Zlatanova, A. Fabbri), Springer*, pp. 407- 426.

Cross, P. A., 1983. Advanced least squares applied to position fixing. *Working Paper No. 6*. Department of Land Surveying, North East London Polytechnic.

Ince, C. D., and Sahin, M., 2000. Real-time deformation monitoring with GPS and Kalman filtering. *Earth Planet Space*, 52, pp. 837-840.

Tasci, L., 2010. Analysis of dam deformation measurements with the robust and non-robust methods. *Scientific Research and Essays*, 5(14), pp. 1770-1779.

Kalman, R. E. (1960). A new approach to linear filtering and prediction problems. *Journal of Basic Engineerig*, 82D, pp. 35-45.

Lim, M. C., Setan, H., Othman, R. and Omar K., 2010. Continuous deformation detection and visualisation of ISKANDARnet. *International Symposium on GPS/GNSS*. October 26-28. Taipei, Taiwan.

Md Som, Z. A., Setan, H. and M Idris, K. N., 2004. A geodetic deformation survey to monitor the behavior of a concrete slab during its axial compression testing. *1st FIG International Symposium on Engineering Surveys for Construction Works and Structural Engineering*. 28 July. Nottingham, United Kingdom.

USACE, 2002. Structural deformation surveying. Department of the Army, US Army Corps of Engineers, EM 1110-2-1009, Washington, D.C.

Welch, G. and Bishop, G., 2001. An introduction to the kalman filter. *Proceeding ACM SIGGRAPH 2001, Course 8*, August 12-17. Los Angeles, CA.

Welch, G. and Bishop, G., 2006. An introduction to the kalman filter. Department of computer science, University of North Carolina at Chapel Hill, TR 95-041, July 24, 2006.